Rheinische Friedrich-Wilhelms-Universität
BONN

DIPLOMARBEIT

Optimal Unemployment Insurance and Administrative Costs

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53111 Bonn

Abgabetermin: 24. Mai 2006

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1 Introduction and Literature Overview

The economics of unemployment insurance seek to explain, positively, how the presence of unemployment insurance institutions, of which the benefit schedule is only one means, among many others, affect the characteristics of unemployment, most importantly its duration. Normatively, the theory aims at designing unemployment insurance institutions that aim at mitigating adverse effects induced by the presence of such institutions. The present diploma thesis shall be a contribution to the normative branch of the economics of unemployment insurance. This branch is commonly referred to as the theory of optimal unemployment insurance.

An unemployment insurance, henceforth abbreviated as UI, is set up to insure workers against the loss of income due to unemployment and to smooth consumption over time and across states of unemployment and employment. Providing more generous systems, however, gives rise to adverse effects such as, for instance, reduced search effort on the side of an unemployed worker and thus prolonged unemployment and higher costs on the side of the UI agency. An optimal UI sets out to strike a balance between insuring an unemployed worker and reducing adverse effects.

The following model is concerned with designing an optimal UI. While providing the worker insurance, the UI agency is confronted with a design problem for two reasons. First, the UI agency's problem is that it cannot observe the unemployed workers' search activities and hence is not able to enforce a given level of search activity which might be optimal to minimize costs of providing UI. This is the problem of moral hazard. Second, funds available to the UI agency not only have to cover benefits but administrative costs as well. Hence, what a worker gives to the UI agency in form of premiums or taxes, he cannot expect to get in return in form of benefits. Expressed differently, the sum of premiums paid does not equal the return in form of benefits even if the insurance were able to observe an insured individual's activities. To conclude this, moral hazard and administrative costs are further reasons why the principal only offers partial coverage.

The model developed in this diploma thesis enriches the models by Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) and studies how the sequence of benefits and taxes should be set when administrative costs enter the picture. The study of administrative costs was suggested in a survey on the optimal UI literature by Karni (1999). Furthermore, I will develop a graphical exposition of the models presented here. Figures will show the intertemporal trade-off between receiving benefits today and wage income tomorrow.

In the following, I will describe their model and how I intend to enrich it. While doing this, I will also discuss crucial differences and similarities to other models.

The problem of insuring an individual is modeled as a repeated moral hazard problem between a risk-neutral UI agency and an infinitely-lived risk-averse individual. Henceforth, UI agency and individual will be called principal and agent, respectively. Time is discrete and begins in period 0. The agent is then unemployed. Being confronted with a UI system set up by the principal, he privately chooses how much costly search effort to put in. Depending on that, he finds employment or stays unemployed in the next period. Baily (1977, 1978) studies the design of an optimal UI in a two-period model. The principal uses a history-independent tax to balance his budget in this model. Flemming (1978) extends Baily's analysis and allows for an infinite time horizon. He focuses on capital market imperfections.

The infinitely-lived agent represents a problem that has always existed and will continue to exist, namely the problem of unemployment. Considering only one agent does not necessarily mean that all individuals are alike. One can look at him representing the rest of the labor force. Or one can think of the agent capturing the aggregate problem of unemployment of a subpopulation. In this sense, the "individual" would be the entire unemployed subpopulation. However, first assuming only one type of individual and abstracting from adverse selection issues considerably simplifies the analysis. Mortensen (1983) is the first who studies the issue of optimal UI and adverse selection, however, in a static environment and abstracting from search incentives. Wang and Williamson (2002) set up a dynamic model with moral hazard and adverse selection focusing on the welfare effects of an experience rated UI. Hopenhayn and Nicolini (2001) study the issue in a two-period model in a framework comparable to their 1997-paper. The principal, however, can observe an agent's type and therefore make the UI contract be contingent on it. Hagedorn, Kaul and Mennel (2002) study a two-type model with a good and a bad searcher in a dynamic world with moral hazard in the spirit of Hopenhayn and Nicolini (1997) and derive properties of a UI contract that separates good and bad searchers.

Moral hazard only takes place on the agent's side. Hopenhayn and Nicolini (1997) assume that the principal cannot observe an agent's search activity. Exerting more search effort positively influences the probability that he receives a job offer guaranteeing him a constant and permanent wage. It he receives a wage offer, he has to accept it. Turning it down is not possible. The UI agency can monitor this decision at no cost. Baily (1977,

1978) and Shavell and Weiss (1979) additionally assume that an unemployed worker can also privately choose his reservation wage and thus making wage offer acceptance another choice variable. This is the point where the theory of job search comes in. This was strongly influenced by Mortensen (1977). In this model, he studies the effect of the presence of a UI delivering a constant stream of benefits over the length of unemployment. The probability of transition from unemployment to employment, called the escape rate, is the probability of getting a job offer, being a function of search effort, and the probability of finding a job, being dependent on the reservation wage. In Shavell and Weiss (1979), putting in more search effort shifts the distribution of wages from which a private wage offer is sampled. Although unrealistic, Hopenhayn and Nicolini (1997) disregard this issue for analytical convenience.

The agent can choose a nonnegative search effort from a continuum. Shavell and Weiss (1979), among many others, assume this as well. For reasons of simplicity and since it does not alter the analysis and the results, Pavoni (2003, 2006) and Hopenhayn and Nicolini (2005) assume that the agent can only choose between searching and not searching. I will, however, follow Hopenhayn and Nicolini (1997) in order to derive comparable results.

Employment is an absorbing state meaning that once the agent has escaped unemployment, he never again faces the problem of unemployment. Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) do not allow for multiple spells of unemployment and so does the following model. Certainly, job performance also affects the probability of losing the job and may thus call for insurance, however, moral hazard on the job is assumed away. Suspending with this assumption, Wang and Williamson (1996, 2002) include transitions in and out of employment with job search and retention effort being unobservable. In the model by Zhao (2000), a worker's job effort also influences the observable output. Here, the principal can observe the entire employment history and hence can design a UI system that is contingent not only on the length of the last spell of unemployment but also on other periods of employment. Hopenhayn and Nicolini (2005) incorporate the termination of employment with an exogenously given probability and show that their prior findings are still valid. Shavell and Weiss (1979, footnote 4) may have conjectured this result when they wrote that "the only effect would be to change [the results and proofs] by a constant 'value of the optimal continuation' given that a job is found." If the job terminates, principal and agent are confronted with the same problem as in the initial period of unemployment. The constant would be the expected discounted costs associated with the optimal contract times the probability of loosing the job.

In this model, we assume that UI benefits and wage income are the only source of income and that the principal can directly control the agent's consumption stream. Hence, income equals consumption in all periods . The direct control assumption is important, because it makes it impossible for the agent to borrow or save without him noticing and to use other vehicles to transfer wealth intertemporally and across states. Contrary to this approach, Flemming (1978) is the first who studies an optimal UI while focussing on capital market imperfections. He shows that there is need of UI because of capital market imperfections since unemployed agents cannot perfectly borrow against future wage claims and have only limited access to capital markets. Shavell and Weiss (1979) study their model under the assumption that the agent has wealth, and thus can save or dissave, or can borrow at the outset of unemployment. However, for the case of unobservable search activity, they cannot characterize the benefit schedule. Although not directly addressing the problem of optimal UI, Cole and Kocherlakota (2001) consider an economy with a risk-neutral principal who insures risk-averse agents facing random income shocks and able to privately save, but not borrow. These two items give rise to the problem of moral hazard and self-insurance. Werning (2002) studies an optimal UI and allows the agent to save or borrow. He finds that the benefit schedule is increasing. The model by Kocherlakota (2004) also works in this environment while focusing on the first-order approach's validity. Finally, Shimer and Werning (2005) show that the benefit schedule for an agent with constant absolute risk aversion is flat if he has access to a riskless asset. The problem with our assumption is that hidden savings may interfere with the UI agency's intention to influence the agent's well-being while unemployed and therefore guide his search activity. If the agent has sufficient liquidity or has access to capital markets, he can borrow against his future income or partially save his benefits in order to smooth consumption. By doing so, he himself provides insurance thus lowering the need for public UI. Without loss of generality, we may also assume that the agent has some other constant, exogenous source of income (see Shavell and Weiss (1979, footnote 5). The pivotal difference is that this type of income is independent of his own activities.

The model by Hopenhayn and Nicolini (1997) assumes commitment to the UI contract on side of the principal and the agent. That means that the principal cannot walk away should costs skyrocket. The agent has to stick to the contract no matter what it yields. In the worst case, he could even face immiseration. Other authors, although not in the context of optimal UI, suspend with this assumption. Thomas and Worral (1988) and Kocherlakota (1996) study an economy where a principal, who has access to a risk-free loan market, insures risk-averse households against unobservable income shocks. Every period, the agent is free to walk away and to live in autarky. The principal's problem is to design a self-enforcing contract so that the agent does not have any incentive to cancel the contract at any point of time. He must include participation constraints that ensure that the agent at least receives the utility of his outside option in every period. In the present model, we will disregard the problem of commitment.

The principal affects the agent's well-being in two ways. First, the principal distributes benefits to him and second, the principal taxes his wage once he receives salary. The last policy instrument in Hopenhayn and Nicolini's model is new; Shavell and Weiss (1979) do not allow for taxation. In their model, once the agent is employed, he is beyond the UI agency's reach. The UI scheme has to provide the agent with an ex-ante given level of utility (this is the insurance aspect). Economic efficiency requires that the agent be better off under the UI system than under no system at all. Otherwise abolishing the UI system would both make the principal and the agent better off which is a Pareto improvement. That is, the UI should provide the agent with a level of utility that is at least as high as the level of utility he receives under no system at all.

As shown by several authors, and documented in survey articles by Atkinson and Micklewright (1991) and Holmlund (1998), the presence of UI adversely affects an agent's job search decision. In order to shorten the spell of unemployment as much as possible and hence minimize the cost of providing UI, the principal has to find a way to make the agent choose search effort optimally so that he escapes unemployment as soon as possible (this is the incentive issue).

Insurance and incentive issue are two conflicting ends. The principal's problem is to strike a balance between those two conflicting ends. He has to find an optimal mix of benefits and taxes that minimize his cost while guaranteeing a given level of utility to the agent. This is the trade-off he faces.

The story so far is told by Hopenhayn and Nicolini (1997). I will now present how to study administrative costs within their framework. One attempt to study them is Raviv (1979) who studies an insurance economy where an agent faces random income shocks in a simple two-period model. He does not consider moral hazard or adverse selection issues, however. In the model, there are two types of administrative costs. First, administrative costs accrue because distributing a larger amount of benefits is not without costs. I assume that benefit distribution costs are higher when benefits are higher. The rationale for this assumption is the following. The principal is required to distribute benefits transparently. If he pays out a larger amount, more officers are required to do the job. They have to check, cross-check and retrace the sums paid which also takes longer do to. Costs to control the payment of benefits increase. Second, taxing wages is not without costs as well. Similarly, tax collection costs rise with the size of the tax revenue. They rise because exacting payment may be more difficult, that is, dunning is more costly and takes longer. More officers or working hours are also necessary to control a larger amount of money.

Including administrative costs in the basic model gives rise to new questions. The optimal sequences of benefits, after-tax income and search effort will change. On the principal's side, how will he set benefits and taxes to cover administrative costs but while giving the agent appropriate incentives to search? On the agent's side, how will the agent choose his search activity in order to adjust to the different setting. Another question that arises in this context is who has to bear the burden of administrative costs. Do benefit claimants receive less coverage or do employed workers have to pay higher taxes? Since we consider two types of administrative costs, the question is whether we can figure out the effects of these two cost centers. Finally, we would like to know more about the impact on total costs. It will then be interesting to study the change in welfare due to administrative costs.

To sum up, my contribution is to look into the design problem of an optimal UI when funds available to the principal also have to cover administrative costs using the framework developed by Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997).

The diploma thesis proceeds as follows. In chapter 2, I will introduce the formal setup of the model. Here, I will define the general Hopenhayn and Nicolini framework and the UI contract. I will discuss administrative costs in more detail. Chapter 3 will turn to the agent's view and study his behavior and outcome in a world without UI where he is forced to live in autarky. Chapters 4 and 5 study the model first without and then with administrative costs, respectively. In the former chapter, I will also focus on the underlying methodology used to build and solve the model. The latter will first show how the basic model is to be modified in order to allow for administrative costs. Then, I will proceed as in the basic model and solve the extended variant. Chapter 6 will conclude the analysis and discuss what to do with the theoretical results derived.

2 Setup of the Model

2.1 The Economy

An infinitely-lived agent is assumed to be a homo oeconomicus that has a publicly known von Neumann-Morgenstern expected utility function:

$$\mathbf{E}\sum_{t=0}^{\infty}\beta^t\big[u(c_t)-e_t\big] \tag{2.1}$$

$\beta < 1$ is the agent's discount factor which, for simplification, stays constant over time. Assuming an infinitely-lived agent avoids specifying what happens after the termination date. In period t, the variable c_t attains the value b_t or w_t when the agent is unemployed and receives benefits or employed and lives from his (net) wage, respectively. $e_t \geq 0$ denotes the search effort. If the agent does not search at all, $e_t = 0$, he does not find employment. If and only if he searches, $e_t > 0$, he finds employment with positive probability. **E** denotes the expectation operator and captures the uncertainty about the future history of employment states and consumption. We have to use this operator because an agent's consumption follows a stochastic process.

This process is governed by a function that gives the probability of finding a job given a level of search activity, $p_t(e_t)$. For convenience and simplification, the probability function remains the same in all periods, hence $p_t(\cdot) = p(\cdot)$. If the agent does not search, we have $p(0) = 0$. It strictly increases for all values of e_t, $p'(e_t) > 0$, and is strictly concave, $p''(e_t) < 0$. It satisfies the Inada conditions, that is, $\lim_{e_t\to 0} p'(e_t) = \infty$ and $\lim_{e_t\to\infty} p'(e_t) = 0$. Thus, the Inada condition brings about that the function $p'(\cdot)$ attains strictly positive values for all nonnegative levels of search effort.

The agent's utility function $u(\cdot)$ strictly increases and is strictly concave, that is, $u'(\cdot) > 0$ and $u''(\cdot) < 0$.[1]

The agent has no other sources of consumption. At the outset of unemployment, neither does he have wealth, nor can he borrow or save. When the agent finds employment, the job offers him a permanent and constant gross wage w. However, he may not dispose of w but of w_t because the principal can control his consumption during employment via a net transfer from him to to the agent.

The principal is also assumed to be a homo oeconomicus who minimizes the following

[1] Disutility from work is not considered since it would be a constant term to be substracted from an employed agent's utility. Qualitatively it does not alter the analysis. However, it may have consequences from a quantitative perspective.

expression:

$$\mathbf{E}\sum_{t=0}^{\infty}\beta^t(c_t - y_t) \tag{2.2}$$

y_t is either w or 0 if the agent is either employed or unemployed, respectively. In periods of unemployment, the principal gives $c_t = b_t$ to the agent and receives $y_t = 0$ in return. As a consequence, the sum increases. Once the agent is employed, the principal receives $y_t = w$ but in turn has to give $c_t = w_t$ to the agent. If the latter is smaller than w, the principal levies a tax $-w + w_t$ on the agent. Otherwise he subsidies the agent's wage. The collection of taxes decreases the sum's value. A wage subsidy increases it. The sum's evolution is governed by the same stochastic process as above. For simplicity, the principal uses the same discount factor β.

2.2 The UI Contract

This section describes the UI contract and the principal's problem. First, I will state his problem and then move on to present the technical subtleties to solve it.

The employment history, up to the beginning of period t, is captured by a publicly known vector $h^t = [h(0), h(1), \ldots, h(t-1)]$. That the employment history h^t is known follows the assumption that the principal directly controls the agent's consumption and that the agent cannot turn down a job offer. $h(t)$ is either 0 or 1 if the agent is unemployed at the beginning or employed at the end of period $t-1$, respectively. Thus, h^t contains $t-1$ zeros if the agent is unemployed at the beginning of period t. If the agent has found employment in period $\tilde{t} < t$, the vector h^t contains $\tilde{t} - 1$ zeros and $t - \tilde{t}$ ones. The employment history is a random object whose occurrence depends on the stochastic process described above. Thus, the probability that the agent is still unemployed at the beginning of period t is $\prod_{k=0}^{t-1}(1 - p(e_k))$ and the probability that the agent is employed at the outset of period t is $\prod_{k=0}^{t-2}(1 - p(e_k))p(e_{t-1})$. To conclude, the random vector h^t describes the agent's employment status at the outset of period t.

At the outset of unemployment in period 0, the principal proposes a UI contract to the agent. For each possible history of employment states, h^t, the contract specifies the stream of consumption, $\{c_t(h^t)\}_{t=0}^{\infty}$, and a recommended effort level, $\{e_t(h^t)\}_{t=0}^{\infty}$. Consumption c_t and effort e_t are functions of the random employment history. That is why we include h^t as the function's argument. If we left the argument, this would refer to c_t as a non-random object.

Denote the contract chosen in period 0 by $\mathcal{W}_0$. Put formally, the contract described above is: $\mathcal{W}_0 = \{c_t(h^t), e_t(h^t)\}_{t=0}^{\infty}$.

Once the agent has found employment in period t, the contract provides him with a constant and permanent wage w_t from period $t+1$ onwards. This follows from the assumption that there is no moral hazard on the job. Then it is best to perfectly smooth consumption over states of employment.

On the principal's side, the contract $\mathcal{W}_0$ results in a stream of positive benefits he distributes and a stream of net transfers from him to the agent that make up this costs. Let $\mathbf{C}(\mathcal{W}_0)$ denote the principal's cost function associated with any contract $\mathcal{W}_0$:

$$\mathbf{C}(\mathcal{W}_0) = \mathbf{E}\sum_{t=0}^{\infty}\beta^t(c_t - y_t) \tag{2.3}$$

On the agent' side, the contract $\mathcal{W}_0$ provides him with consumption and prescribes how hard to search which make up his life-time utility. Let $\mathbf{V}(\mathcal{W}_0)$ denote his expected discounted utility associated with any contract $\mathcal{W}_0$:

$$\mathbf{V}(\mathcal{W}_0) = \mathbf{E}\sum_{t=0}^{\infty}\beta^t\big[u(c_t) - e_t\big] \tag{2.4}$$

Given this specification, we are now prepared to state the principal's problem. At the outset of unemployment, he wants to choose a contract $\mathcal{W}_0$ that minimizes his cost function, $\mathbf{C}(\mathcal{W}_0)$, but promises the agent a value $\mathbf{V}(\mathcal{W}_0)$ that is at least as high as an ex-ante given utility V_0^u. Let $C(V_0^u)$ be the value of the principal's optimal contract that promises an ex-ante given value V_0^u to the agent. Put formally, the principal's problem is:

$$C(V_0^u) = \min_{\mathcal{W}_0}\mathbf{C}(\mathcal{W}_0) \tag{2.5}$$

subject to

$$\mathbf{V}(\mathcal{W}_0) \geq V_0^u \tag{2.6}$$

The constraint (2.6) the principal has to observe is called the promise-keeping constraint. It asserts that the contract $\mathcal{W}_0$ gives the agent at least an expected discounted utility of V_0^u. We may not mistake this constraint for a participation constraint. We need to impose the promise-keeping constraint only at the beginning of period 0. A participation constraint ensures that the agent receives at least the utility level from his outside option and thus brings about self-enforcing of the contract. In this sense, the promise-keeping constraint is a special participation constraint that ensures participation in period $t = 0$. Why we disregard participation constraints and impose only a promise-keeping constraint has several reasons. From a technical point of view, not taking into consideration further constraints makes the problem easier to deal with. However, this then has economic implications. In the present setting, only at the outset of unemployment the agent decides whether to enter the contract or to live in autarky. Afterwards, he is not allowed to make this decision anymore and must adhere to the contract's conditions. To conclude, the nature of promise-keeping and participation constraints comes from the problem of commitment and enforcement. We assume commitment to and enforcement of the contract. The contract needn't be self-enforcing.

The optimal UI contract is studied under two assumptions about the principal's capability to observe an agent's search activity. First, I will assume that the principal can observe an agent's search effort and thus enforce an optimal level. In this environment, the agent cannot engage in hidden actions and thus give rise to the problem of moral hazard. This is the case of perfect information. The literature refers to the solution as the first-best solution. Although this case is rather realistic, considering it has advantages. First, from the principal's point of view, it would be best if he had control over the agent's search activity decision in all periods in order to minimize his costs. Then we can compare other scenarios to it, in particular to those where the principal only has imperfect information over the agent's activities, and thus serves as a benchmark. Second, by abstracting from incentive considerations, the perfect information environment allows to study insurance aspects of a UI exclusively. In the second case, I assume that the principal cannot observe an agent's search effort and thus cannot enforce it. The result is called the second-best solution. Since, from a cost-minimizing point of view, it is best that the agent always searches, the principal has to devise a mechanism that gives the agent appropriate incentives to search. Since the agent reacts differently to different contracts proposed, the principal has to take this behavior into consideration. The contract he proposes to the agent has to be compatible with the incentive that makes the agent optimally search in every period. Thus, in an imperfect information environment, the principal faces another constraint, the incentive-compatibility constraint.

2.3 Administrative Costs

I assume that distributing benefits or collecting taxes is not without costs. Distributing more benefits or collecting more taxes involves higher costs and becomes increasingly costly. I will abstract from fixed costs because they would not alter the qualitative analysis although they would affect a quantitative one. As the principal's problem and his cost function are modeled, benefits and benefit distribution costs enter the cost function positively. Taxes and tax collection costs enter it negatively and positively, respectively.

Let $a(\cdot)$ denote the administration cost function. The cost function has the property $a(0) = 0$, no fixed costs. On the positive part of the abscissa, $a(\cdot)$ strictly increases, $a'(x) > 0$ for $x > 0$, that is, higher benefits involve higher benefit distribution costs. On the negative part of the abscissa, $a(\cdot)$ strictly decreases, $a'(x) < 0$ for $x < 0$, that is, higher taxes, a smaller value of x, involve higher tax collection costs. The cost function is convex on both parts of the abscissa. We further assume that $\lim_{x \to 0} a'(x) = 0$. A simple function that satisfies these requirements is $x \mapsto x^2$. It needn't necessarily be symmetric to the ordinate, however.

2.4 Variants of the Model

To study the model, I will develop it successively. I will add feature after feature to the model, take one away but include another and so forth. By doing so I will exhaust all possible combinations and slowly show how each feature changes the analysis and how they interact with each other. In the end, I will take all features and include them in the model. The features are the following: taxation of wages, benefit distribution costs and tax collection costs. I will start to analyze a model without administrative costs. This will be one developed by Hopenhayn and Nicolini (1997). After that, I will study variants of this model that feature benefit distribution costs, tax collection costs and finally both cost structures. Each of the four variants is studied under the assumption of perfect and imperfect information.

3 Autarky Problem

This section studies an agent's behavior who does not have access to UI. The study serves as a benchmark and derives a lower bound of a newly unemployed agent's expected discounted utility.

"Nature" proposes the following contract $\mathcal{W}_0$ to him. During unemployment, the agent does not consume anyhing in each period t, $c_t = 0$. When he finds a job in period t, he consumes his gross wage $c_t = w$, which is his net wage since no institution taxes his income, from period $t+1$ onwards . Since no UI agency recommends a search activity, the agent has to choose a sequence $\{e_t(h^t)\}_{t=0}^{\infty}$ that maximizes his expected discounted utility from being unemployed as given by (2.1) in period 0. Let $\mathbf{V}(\mathcal{W}_0)$ denote his expected discounted utility associated with any contract $\mathcal{W}_0$, that is, any sequence of search effort chosen in period 0. The agent aims at finding an optimal contract $\mathcal{W}_0^*$ that maximizes $\mathbf{V}(\mathcal{W}_0)$. Let V_0^u denote the agent's expected discounted utility that is associated with his optimal search activity decisions:

$$V_0^u = \max_{\{e_t(h^t)\}_{t=0}^{\infty}} \left\{ \mathbf{V}(\mathcal{W}_0) = \mathbf{E} \sum_{t=0}^{\infty} \beta^t \big[u(c_t) - e_t\big] \right\}$$

This is the agent's dynamic programming problem. Given a contract $\mathcal{W}_0$, let $\mathcal{W}_t$ denote the contract continuation in period t. That is, $\mathcal{W}_t$ specifies contract $\mathcal{W}_0$'s actions from period t onwards. Given this notation, we can expand the expression in the curly brackets:

$$V_0^u = \max_{e_0, \mathcal{W}_1} \big\{ \mathbf{V}(\mathcal{W}_0) = u(0) - e_0 + \beta\big[p(e_0)V_0^e + (1 - p(e_0))\mathbf{V}(\mathcal{W}_1)\big] \big\} \tag{3.1}$$

where

$$V_t^e = \frac{u(w)}{1 - \beta} \quad \forall t \geq 0$$

V_t^e designates an employed agent's wage income, discounted to period $t+1$. Henceforth, we call V_t^e the continuation value of employment. It captures an employed agent's wealth from period $t+1$ onwards. Since V_t^e is the same in all periods, we can leave the time index and equally work with V^e.

So far we have split the agent's decision in choosing e_0 optimally "today" and the optimal plan continuation $\mathcal{W}_1$ "tomorrow". The plan continuation $\mathcal{W}_1$ solves the following problem:

$$V_1^u = \max_{\{e_t(h^t)\}_{t=1}^{\infty}} \left\{ \mathbf{V}(\mathcal{W}_1) = \mathbf{E} \sum_{t=1}^{\infty} \beta^t \big[u(c_t) - e_t\big] \right\}$$

The optimal plan continuation $\mathcal{W}_1^*$ yields $V_1^u = \mathbf{V}(\mathcal{W}_1^*)$. This result enters problem (3.1) as an additional constraint. It now reads:

$$V_0^u = \max_{e_0} \left\{ u(0) - e_0 + \beta\big[p(a_0)V^e + (1 - p(e_0))V_1^u\big]\right\}$$

This is the Bellman equation for V_0^u. In all periods t, the agent does not receive any benefits and the wage stays constant. Thus, if we shifted our problem in period 0 by one period into the future, the problem would look the same in period 1. The continuation value of unemployment stays constant and we have $V_0^u = V_1^u$. Moreover, since the problem is the same in all periods, we have $V_t^u = V_{t+1}^u$, it is stationary. Then we can drop the time index. Hence, the problem in any period t becomes:

$$V^u = \max_{e} \left\{ u(0) - e + \beta\big[p(e)V^e + (1 - p(e))V^u\big]\right\}$$

Dynamic optimality requires the agent to make an optimal decision "today" given that he will make optimal decisions "tomorrow" being contingent on "today's" decisions. The present problem's structure shows that the agent only has to choose his search effort once because what solves his problem optimally "today" will also yield an optimal solution to "tomorrow's" problem. The first-order condition for this problem is:

$$\beta p'(e^*)\big(V^e - V^u\big) = 1 \tag{3.2}$$

if the agent chooses $e^* > 0$. Since, in this problem, there is no state variable that evolves over time, the condition gives an time-invariant optimal search effort decision and an associated value of being unemployed labeled V_{aut} - the autarky value of being unemployed. It is defined as:

$$V_{\text{aut}} = V^e - \frac{1}{\beta p'(e^*)} \tag{3.3}$$

If the principal is interested in improving the fate of an unemployed agent, he should design a UI that gives him an expected discounted value of unemployment at the outset that is at least as high as the autarky value. Introducing a UI makes him better off, thus, this is socially efficient.

In the sequel, we will return to condition (3.2). Given the continuation values of employment and unemployment, the agent makes an optimal search decision in the current period. Hence, if the principal cannot tell the agent to exert a prescribed search effort, he can set the two variables so that the agent reacts by setting his search effort as he wished him to do.

4 Model without Administrative Costs

4.1 Introduction and Methodology

In this section, I present Hopenhayn and Nicolini's model that extends Shavell and Weiss's model insofar as it will now allow for taxation of wages. The principal may influence the continuation value of employment.

Recall the basic optimization problem (2.5) subject to the promise-keeping constraint (2.6). In order to allow for taxation, the principal is now assumed to be able to control the agent's stream of consumption once he is employed.

$\mathcal{W}_0 = \{c_t(h^t), e_t(h^t)\}_{t=0}^{\infty}$ denotes the contract the principal chooses at the outset of unemployment in period 0 and $\mathcal{W}_t$ denotes the contract continuation of $\mathcal{W}_0$ from period t onwards. Now, an employed agent may not dispose of w but of w_t from period $t+1$ onwards. Since no incentive issues arise on the job, it is best to give the agent a constant consumption pattern. When the agent receives w_t, there is a net transfer from the principal to the agent $-w + w_t$. If $w > w_t$, the net transfer to the agent is negative, hence it is tax. If it is positive, the principal subsidies the agent. The principal's transfer, discounted to period $t+1$ is:

$$W(w_t) = \frac{-w + w_t}{1-\beta}$$

The principal's cost function associated with any contract $\mathcal{W}_0$ is:

$$\mathbf{C}(\mathcal{W}_0) = b_0 + \beta\Big[p(e_0)W(w_0) + (1-p(e_0))\mathbf{C}(\mathcal{W}_1)\Big] \tag{4.1}$$

where $\mathbf{C}(\mathcal{W}_1)$ denotes expected discounted cost associated with contract continuation of the contract $\mathcal{W}_0$ chosen in period 0.

The agent's expected discounted utility associated with any contract $\mathcal{W}_0$ is given by:

$$\mathbf{V}(\mathcal{W}_0) = b_0 + \beta\Big[p(e_0)\frac{u(w_0)}{1-\beta} + (1-p(e_0))\mathbf{V}(\mathcal{W}_1)\Big] \tag{4.2}$$

Let $C(V_0^u)$ denote the principals cost function induced by an optimal contract $\mathcal{W}_0$ that promises the agent an ex-ante given utility level V_0^u. Formally, we have:

$$C(V_0^u) = \min_{\mathcal{W}_0} \left\{ \mathbf{C}(\mathcal{W}_0) = \mathbf{E} \sum_{t=0}^{\infty} \beta^t (c_t - y_t) \right\} \tag{4.3}$$

subject to

$$\mathbf{V}(\mathcal{W}_0) = \mathbf{E} \sum_{t=0}^{\infty} \beta^t \big[u(c_t) - e_t\big] \geq V_0^u \tag{4.4}$$

and

$$V_0^u \text{ given.} \tag{4.5}$$

Now expand expressions (4.3) and (4.4):

$$C(V_0^u) = \min_{b_0, w_0, e_0, \mathcal{W}_1} \Big\{ \mathbf{C}(\mathcal{W}_0) = b_0 + \beta \Big[p(e_0) W(w_0) + (1 - p(e_0)) \mathbf{C}(\mathcal{W}_1) \Big\} \tag{4.6}$$

subject to

$$\mathbf{V}(\mathcal{W}_0) = u(b_0) - e_0 + \beta \big[p(e_0) \frac{u(w_0)}{1 - \beta} + (1 - p(e_0)) \mathbf{V}(\mathcal{W}_1) \big] \geq V_0^u \tag{4.7}$$

We have split the principal's problem of choosing an optimal contract $\mathcal{W}_0$ into choosing b_0, w_0 and recommended search effort e_0 "today", and the optimal contract continuation $\mathcal{W}_1$ "tomorrow".

Instead of choosing w_0 and w_t for all $t \geq 1$, we will work with a monotonic transformation. Call V_t^e the continuation value of employment. It gives the agent's utility, discounted to period $t+1$, from being employed:

$$V_t^e = u(w_t) + \beta u(w_t) + \beta^2 u(w_t) + \ldots = \frac{u(w_t)}{1 - \beta}$$
$$\Leftrightarrow w_t = u^{-1}\big((1 - \beta) V_t^e\big)$$

Both w_t and V_t^e equivalently describe an employed agent's wealth position. By using the transformation, we can reformulate the principal's discounted value of transfers and state it in terms of V_t^e:

$$W(V_t^e) := \frac{-w + w_t}{1-\beta} = \frac{-w + u^{-1}\big((1-\beta)V_t^e\big)}{1-\beta}$$

The function $W(V_t^e)$ strictly increases and is strictly convex in V_t^e. The derivatives are:[2]

$$W'(V_t^e) = \frac{1}{u'(w_t)} > 0, \quad W''(V_t^e) = -\frac{u''(w_t)(1-\beta)}{\big(u'(w_t)\big)^3} > 0$$

Therefore, in problem (4.6), we now choose V_0^e instead of w_0, or generally, V_t^e instead of w_t in all periods $t \geq 0$.

The trick now is to give another expression for $\mathbf{C}(\mathcal{W}_1)$ given that the contract continuation $\mathcal{W}_1$ is part of an optimal contract $\mathcal{W}_0$ chosen in period 0. The technique used is referred to as the theory of recursive contracts and relies on the Bellman principle.

Denote the optimal contract continuation by $\mathcal{W}_1^*$ and by $\mathbf{V}(\mathcal{W}_1^*) = V_1^u$ the continuation utility it yields to the agent. If in period 1 the principal has to implement a UI contract that promises V_1^u to the agent, he solves the following problem:

$$C(V_1^u) = \min_{\mathcal{W}_1} \mathbf{C}(\mathcal{W}_1) \quad \text{subject to} \quad \mathbf{V}(\mathcal{W}_1) \geq V_1^u \tag{4.8}$$

Whatever the continuation of the optimal contract $\mathcal{W}_0$ chosen in period 0 may be, it cannot yield lower costs than the optimal contract that delivers V_1^u, that is, solves the problem defined by (4.8). Hence, we have $\mathbf{C}(\mathcal{W}_1^*) \geq C(V_1^u)$. We get an even stronger result that becomes useful below:

Proposition 1. *If in period 0 the principal has chosen an optimal contract $\mathcal{W}_0^*$, the contract continuation $\mathcal{W}_1^*$ satisfies $\mathbf{C}(\mathcal{W}_1^*) = C(V_1^u)$. In general, for all periods $t > 1$ we have that the contract continuation $\mathcal{W}_t^*$ satisfies $\mathbf{C}(\mathcal{W}_t^*) = C(V_t^u)$.*

Proof. The proof follows Pavoni (1999) and was inspired by Spear and Srivastava (1987). The proof goes by induction. The hypotheses is that $\mathcal{W}_0^*$ has been optimally chosen. The base step goes by contradiction while assuming existence of the statistic V_1^u. Suppose we have $\mathbf{C}(\mathcal{W}_1^*) > C(V_1^u)$. Call the solution to problem (4.8) $\mathcal{W}_1^{**}$. That is, $\mathcal{W}_1^{**}$ and not $\mathcal{W}_1^*$ is the optimal continuation of the contract chosen in period 0. Hence, the contract continuation $\mathcal{W}_1^{**}$ guarantees the agent a promised level of utility V_1^u. The principal could also choose $b_0^{**} = b_0^*$, $V_0^{e*} = V_0^{e**}$ and $e_0^{**} = e_0^*$ and would not violate the period 0-promise-keeping-constraint. This implies, however, that in period 0 the principal should

[2] See appendix A for the derivation of these results.

have chosen $\mathcal{W}_1^{**}$ and not $\mathcal{W}_1^*$ as the optimal contract continuation. It was feasible in period 0 and yields lower costs. This contradicts the assumption that contract $\mathcal{W}_0^*$ was optimal. Therefore, an optimal contract continuation cannot deliver $\mathbf{C}(\mathcal{W}_1^*) > C(V_1^u)$. This establishes the base step.

It follows the induction step. Suppose the contract $\mathcal{W}_{k-1}^*$ has been optimally chosen in period $k-1$ and the contract continuation $\mathcal{W}_k^*$ satisfies $\mathbf{C}(\mathcal{W}_k^*) = C(V_k^u)$ while assuming existence of the statistics V_k^u. We want to show that if the contract continuation $\mathcal{W}_k^*$ has been optimally chosen, then we must have $\mathbf{C}(\mathcal{W}_{k+1}^*) = C(V_{k+1}^u)$. The induction step's proof follows the same line of reasoning as above. Suppose we have $\mathbf{C}(\mathcal{W}_{k+1}^*) > C(V_{k+1}^u)$. This means that some other contract $\mathcal{W}_{k+1}^{**}$ must have solved $C(V_{k+1}^u) = \max_{\mathcal{W}_{k+1}} \mathbf{C}(\mathcal{W}_{k+1})$. Hence, $\mathcal{W}_{k+1}^{**}$ satisfies the period $k+1$ promise-keeping constraint. The principal then should choose $\{c_t^*(h^t)\}_{t=0}^k = \{c_t^{**}(h^t)\}_{t=0}^k$, $\{e_t^*(h^t)\}_{t=0}^k = \{e_t^{**}(h^t)\}_{t=0}^k$. Contracts $\mathcal{W}_0^*$ and $\mathcal{W}_0^{**}$ are equivalent up to period k. From period $k+1$ onwards, we have found a contract continuation $\mathcal{W}_{k+1}^{**}$ that yields lower costs that the contract continuation $\mathcal{W}_{k+1}^*$. This implies the principal should have chosen $\mathcal{W}_{k+1}^{**}$ as the optimal contract continuation. This contradicts the assumption that the principal has optimally chosen contract $\mathcal{W}_k^*$. This establishes the induction step.

By the principle of induction, we must have $\mathbf{C}(\mathcal{W}_t^*) = C(V_t^u)$ for all periods t. This concludes the proof. Q.E.D.

In order to use the Bellman principle, the problem is finding an appropriate state variable. The proposition above shows that we can use the continuation lifetime utility an optimal contract continuation yields as a state variable. We need no longer to specify a complex history-dependent contract. All relevant aspects of the agent's future history are captured by the one-dimensional object "promised value". It plays the role of a state variable. When we now solve the period 0-problem, we also optimize over the choice of the promised value for period 1-problem which in turn induces a constraint on period 0-problem. "Today" we solve "today's" problem by also determining the promised value for "tomorrow's" problem given that "tomorrow" we will solve "tomorrow's" problem subject to the promised value computed "today". By repeatedly doing so, we achieve 'sequential efficiency'. The principal concentrates on the plan's details "today", considering an optimal position for "tomorrow" given that he will also set up an optimal plan "tomorrow". Recursivity now means that the values b_t, e_t, V_t^e and V_{t+1}^u are functions of "today's" promised value of utility V_t^u.

The structure of the principal's problem remains the same in all periods. It is a stationary one. In any period t, he has to choose b_t, e_t, the continuation value of employment V_t^e and the continuation value of unemployment V_{t+1}^u given a promised level of utility V_t^u. I therefore state the principal's problem in period t:

$$C(V_t^u) = \min_{b_t, e_t, V_t^e, V_{t+1}^u} b_t + \beta\big[p(e_t)W(V_t^e) + (1 - p(e_t))C(V_{t+1}^u)\big] \tag{4.9}$$

subject to the promise-keeping constraint

$$u(b_t) - e_t + \beta\big[p(e_t)V_t^e + (1 - p(e_t))V_{t+1}^u\big] \geq V_t^u \tag{4.10}$$

and the initial condition

$$V_t^u \text{ given.} \tag{4.11}$$

The promise-keeping constraint (4.10) is the problem's law of motion that controls the evolution of the promised value of utility.

4.2 Perfect Information

In the perfect information environment, the principal controls the agent's search activity and his consumption while unemployed and employed. His problem is to minimize (4.9) subject to the promise-keeping constraint (4.10) and the initial condition (4.11). The initial condition in period 0 shall be $V_0^u \geq V_{\text{aut}}$. We want to provide the agent with a larger expected discounted utility than the autarky value.

The cost function $C(V_t^u)$ is strictly convex. Providing a larger expected discounted value of unemployment, V_0^u, increases consumption during unemployment and thus decreases the marginal utility. As a consequence, giving the agent an additional "util" can only be achieved at increasing marginal costs in terms of the consumption good (Ljungqvist and Sargent, 2004, p. 754). Hence, the optimization problem is well-behaved and the solutions given by the first-order conditions solve it. The period t-Lagrangian is:

$$\begin{aligned}\mathcal{L} = -\Big\{b_t + \beta\big[p(e_t)W(V_t^e) + (1 - p(e_t))C(V_{t+1}^u)\big]\Big\} \\ + \lambda_t\Big\{u(b_t) - e_t + \beta\big[p(e_t)V_t^e + (1 - p(e_t))V_{t+1}^u\big] - V_t^u\Big\}\end{aligned} \tag{4.12}$$

The first-order conditions with respect to e_t, b_t, V_t^e, V_{t+1}^u and the envelope condition are:

$$\beta p'(e_t^*)\big(W(V_t^{e*}) - C(V_{t+1}^{u*})\big) + \lambda_t\Big(\beta p'(e_t^*)\big(V_t^{e*} - V_{t+1}^{u*}\big) - 1\Big) = 0 \tag{4.13}$$

$$\lambda_t = \frac{1}{u'(b_t^*)} = W'(V_t^{e*}) = \frac{1}{u'(w_t^*)} = C'(V_{t+1}^{u*}) = C'(V_t^{u*}) \tag{4.14}$$

The last two conditions of (4.14) and the cost function's convexity imply $V_t^{u*} = V_{t+1}^{u*}$. Hence, benefits stay constant over time. Moreover, we have $w_t^* = b_t^*$. Consumption is fully smoothed over time and across states. The agent's continuation values of employment and unemployment stay constant over time. The agent is fully insured against the risk of unemployment and the principal bears all the risks. This is the result of risk-sharing between a risk-neutral principal and a risk-averse agent. To give some intuition behind this result, suppose the principal provided the agent with a random wage $\tilde{w}$ with average $\overline{\tilde{w}}$. The agent's utility from $\overline{\tilde{w}}$ would be higher than the expected utility from $\tilde{w}$ because of the agent's utility function's concavity. The principal is indifferent between paying random wage $\tilde{w}$ and constant wage $\overline{\tilde{w}}$. Then economic efficiency requires giving the agent a constant wage $\overline{\tilde{w}}$ because he is better off and the principal is not worse off - a Pareto improvement.

We now consider the principal's search effort decision. We have found out that it is optimal to provide the agent with a constant stream of consumption. Then, in order to give the agent a constant level of utility in each period, the principal prescribes the agent to exert a constant search level e_t. If the principal chose a different level, he would have to adjust benefits or the tax. Suppose the principal deviates from the constant effort requirement in one period. The benefits and wage income would then have to adjust and change. This would violate the optimality condition of constant consumption.

Given these findings, from now on, we drop the time index t and work with $w_t = b_t = b = \overline{w}$, $e_t = e$, $\lambda_t = \lambda$ and $V_t^u = V^u$.

In order to find the costs minimizing e^*, we can use our findings to give an expression to the cost function:

$$C(V^u) = \frac{b^*}{1-\beta} - \frac{\beta p(e)w}{(1-\beta)(1-\beta(1-p(e)))}$$

We can also rearrange the promise-keeping constraint:

$$V^u = \frac{u(\overline{w}^*)}{1-\beta} - \frac{e}{1-\beta(1-p(e))}$$

The Lagrangian for choosing a cost minimizing e is therefore (where I have already substituted for λ):

$$\mathcal{L} = \frac{1}{u'(\overline{w}^*)}\left[-\frac{u'(\overline{w}^*)b^*}{1-\beta} + \frac{\beta p(e) u'(\overline{w}^*) w}{(1-\beta)(1-\beta(1-p(e)))} + \frac{u(\overline{w}^*)}{1-\beta} - \frac{e}{1-\beta(1-p(e))}\right]$$

To maximize over e, we only have to consider those expression where this variable actually appears. Hence, e^* maximizes the following expression:

$$e^* = \arg\max_e \sum_{t=0}^{\infty} \big(\beta(1-p(e))\big)^t \left[p(e)\frac{\beta}{1-\beta}u'(\overline{w}^*)w - e\right] \tag{4.15}$$

One can see that an increasing $\overline{w}^*$ decreases e^*. If the agent's consumption is higher, he receives a higher utility. This, however, implies higher total costs for the principal. The principal chooses e^* to maximize "the expected discounted value of wages net of the cost of search effort." (Hopenhayn and Nicolini, 1997, p. 417)

Let us further study the optimal search effort decision. Above we have also directly derived the condition for a cost-minimizing e, see condition (4.13). It equates the marginal cost savings from search and the agent's marginal gain. Since the first term on the left hand side is positive, the second one must be negative. This term can also be described as the agent's net gain. Hence, the principal's decision imposes a cost on the agent. If the principal were not able to observe the agent's activities, the agent would choose his search activity so that his net marginal cost is zero by lowering his effort. Therefore, the principal's choice of effort is not incentive-compatible.

We can graphically study the present environment. At the optimal triple $(b^*, \overline{w}^*, e^*)$, the first-order conditions (4.14) imply:

$$\frac{u'(b^*)}{u'(\overline{w}^*)} = 1 \tag{4.16}$$

The optimal triple $(b^*, \overline{w}^*, e^*)$ always has to satisfy condition (4.16) in all periods t. We can say more about this equality. Look at the agent's promise keeping constraint (4.10) in period t and consider an actuarially fair change of b and $\overline{w}$. Hence, totally differentiate this expression with respect to b and $\overline{w}$ while holding V^u and e constant:

$$\begin{aligned} \frac{u'(b)}{u'(\overline{w})} &= -\frac{d\overline{w}}{db}p(e)\frac{\beta}{1-\beta} \\ \Leftrightarrow \text{MRS} := \frac{d\overline{w}}{db} &= -\frac{u'(b)}{u'(\overline{w})}\left(p(e)\frac{\beta}{1-\beta}\right)^{-1} \end{aligned} \tag{4.17}$$

Equation (4.17) denotes the agent's intertemporal marginal rate of substitution between receiving benefits b in any period t and wage income $\overline{w}$ from any period $t+1$ onwards which is the slope of the agent's indifference curve that yields utility V^u. The MRS computes by how much after-tax income $\overline{w}$ has to fall in each period from $t+1$ onwards in order to provide one more unit of benefits b while keeping everything else constant, in particular promised utility V^u.

Similarly for the principal, look at his cost function (4.9) in period t and consider a cost-neutral change of b and $\overline{w}$. Hence, totally differentiate this expression with respect to b and $\overline{w}$ while holding V^u and e constant::

$$\begin{aligned} 1 &= -\frac{d\overline{w}}{db} p(e) \frac{\beta}{1-\beta} \\ \Leftrightarrow \text{MRT} := \frac{d\overline{w}}{db} &= -\left(p(e)\frac{\beta}{1-\beta}\right)^{-1} \end{aligned} \tag{4.18}$$

The marginal rate of transformation tells us how much $\overline{w}$ has to fall in order to finance one more unit of b while keeping everything else constant, in particular total costs constant $C(V^u)$.

From inspection of equation (4.16), we see that it is equivalent to the familiar condition MRS = MRT at the optimal point. The environment is depicted in figure 1 on page 21-a. b^* and $\overline{w}^*$ lie on the 45-degree line. The line of transformation is straight. The figure shows that deviating from the optimal triple involves higher costs while the agent's utility stays constant. Moreover, it violates the risk-sharing requirement. This cannot be optimal.

4.3 Imperfect Information

We now study the principal's problem of designing an optimal UI scheme when he cannot observe the agent's search activity. Compared to the full-information environment, it now can no longer be optimal to provide the agent with a constant stream of consumption over time and across states of unemployment and employment because then the agent would stop looking for a job. Hence, the constant-consumption profile provides insufficient incentives to search.

We will harness the techniques as summarized in proposition 1 and write down the principal's problem in recursive form. It is to minimize (4.9) subject to the promise-keeping constraint (4.10), given the initial condition (4.11), and the incentive-compatibility

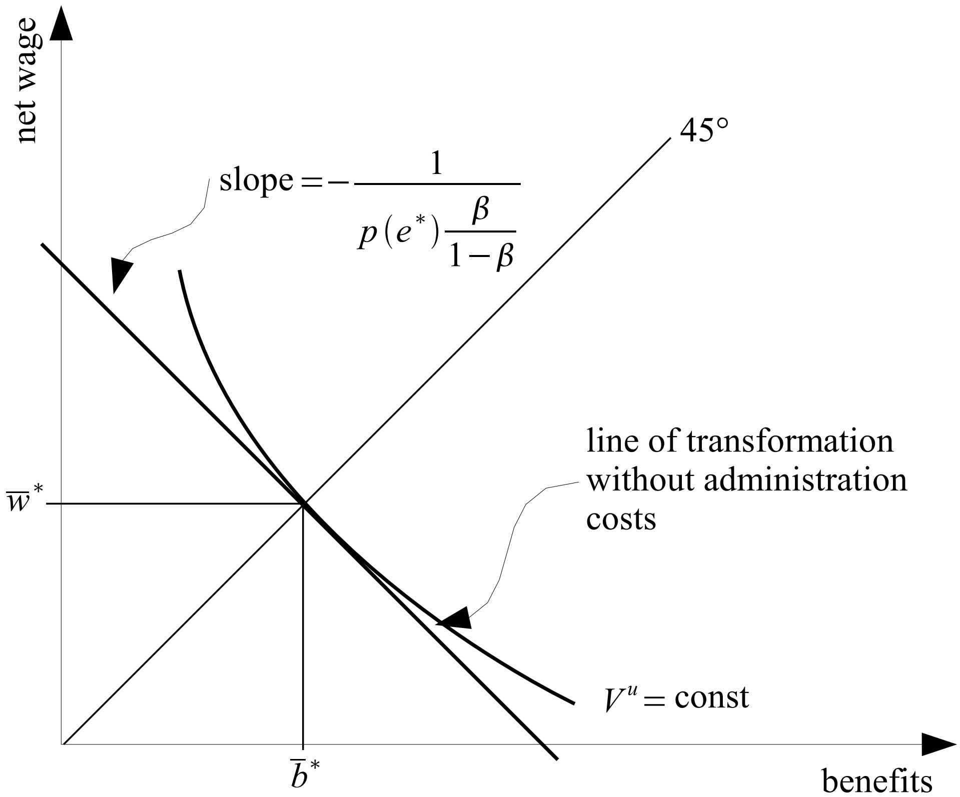

Figure 1: Illustrates the model without administrative costs under perfect information.

constraint

$$e_t \in \arg\max_{\hat{e}_t} u(b_t) - \hat{e}_t + \beta\big[p(\hat{e}_t)V_t^e + (1 - p(\hat{e}_t))V_{t+1}^u\big] \tag{4.19}$$

Working with this expression is almost impossible because it is really an infinite number of constraints. That is why we want to find another expression that is equivalent to (4.19) and easier to handle. It turns out that the first-order condition is the expression sought. It is:

$$\beta p'(e_t)\big[V_t^e - V_{t+1}^u\big] = 1 \tag{4.20}$$

The condition equates the marginal gain from search and the marginal cost from search. The agent puts in more effort until the marginal gain equals marginal cost. The agent's search activity decision depends on the sign of $\big[V_t^e - V_{t+1}^u\big]$. The agent chooses $e_t > 0$ if and only if this sign is positive. Intuitively, the agent searches if and only if he gains from search, otherwise it is better staying unemployed.

Condition (4.20) is only a necessary condition for a maximum. But in order to replace (4.19) by (4.20), it also has to be sufficient. This is the point where the probability function's conditions, concavity and Inada conditions, come into play. Since the probability function is strictly concave and differentiable, the agent's expected discounted utility function is strictly concave and differentiable as well. For all levels of e_t, the derivative of the probability function attains strictly positive values and decreases throughout. Given that the gain from search is positive, there will be an $e_t > 0$ so that the first-order condition is satisfied for all positive levels of $(V_t^e - V_{t+1}^u)$. Given the concavity, what solves the first-order condition also solves the initial problem. Hence, it is also a sufficient condition and we can replace incentive-compatibility constraint (4.19) by its first-order condition (4.20). That is why this way is also referred to as the first-order approach. Again, it is only valid because the concavity of the probability function and the Inada conditions ensure that the necessary first-order condition is also a sufficient one.

Before we start optimizing, we have to say something about the cost function's shape. In order to attain a unique global minimum, it has to be strictly convex. Restrictions (4.10) and (4.20), however, are not linear in the control variables . In this case, the cost function can turn out not to be convex and we would have to impose further restrictions that make the dynamic programming problem convex or develop other techniques to solve the non-convex dynamic optimization problem[3]. Hopenhayn and Nicolini (1997) also solve

[3] Phelan and Townsend (1991) propose using lotteries over the control variables.

their model numerically and find that the cost function turns out to be strictly convex. To derive analytical results and not to focus on subtle restrictions or other techniques, we therefore proceed under the assumption that the cost function is convex grounded on the numerical analysis.

The Lagrangian in period t is:

$$\begin{aligned}\mathcal{L} = &-\Big\{b_t + \beta\big[p(e_t)W(V_t^e) + (1-p(e_t))C(V_{t+1}^u]\big)\Big\}\\ &+ \lambda_t\Big\{u(b_t) - e_t + \beta\left[p(e_t)V_t^e + (1-p(e_t))V_{t+1}^u\right] - V_t^u\Big\}\\ &\qquad + \eta_t\Big\{\beta p'(e_t)\big(V_t^e - V_{t+1}^u\big) - 1\Big\} \end{aligned} \tag{4.21}$$

The first-order conditions with respect to e_t, b_t, V_t^e and V_{t+1}^u and the envelope condition are, respectively:

$$p'(e_t^*)\big(W(V_t^{e*}) - C(V_{t+1}^{u*})\big) = \eta_t p''(e_t^*)\big(V_t^{e*} - V_{t+1}^{u*}\big) \tag{4.22}$$

$$\lambda_t = \frac{1}{u'(b_t^*)} \tag{4.23}$$

$$\lambda_t = C'(V_{t+1}^{u*}) + \eta_t \frac{p'(e_t^*)}{1-p(e_t^*)} \tag{4.24}$$

$$\lambda_t = W'(V_t^{e*}) - \eta_t \frac{p'(e_t^*)}{p(e_t^*)} \tag{4.25}$$

$$\lambda_t = C'(V_t^{u*}) \tag{4.26}$$

The envelope condition can be rewritten as:

$$C'(V_t^{u*}) = (1-p(e_t^*))C'(V_{t+1}^{u*}) + p(e_t^*)W'(V_t^{e*}) = \frac{1}{u'(b_t^*)} \tag{4.27}$$

$$= p(e_t^*)\big(W'(V_t^{e*}) - C'(V_{t+1}^{u*})\big) + C'(V_{t+1}^{u*}) \tag{4.28}$$

For the following analysis, Hopenhayn and Nicolini (1997, p. 435) prove that the Lagrange multiplier η_t is positive. Rearranging first-order conditions (4.24) and (4.25) yields:

$$W'(V_t^{e*}) - C'(V_{t+1}^{u*}) = \eta_t p'(e_t^*)\left[\frac{1}{1-p(e_t^*)} + \frac{1}{p(e_t^*)}\right] \tag{4.29}$$

Thus, we have $W'(V_t^{e*}) > C'(V_{t+1}^{u*})$ and equation (4.28) furthermore implies:

$$W'(V_t^{e*}) > C'(V_t^{u*}) > C'(V_{t+1}^{u*}) \tag{4.30}$$

We finally arrive at:

$$\frac{1}{u'(w_t^*)} > \frac{1}{u'(b_t^*)} > \frac{1}{u'(b_{t+1}^*)} \tag{4.31}$$

The utility function's concavity produces $w_t^* > b_t^* > b_{t+1}^*$. Consumption is not fully smoothed over time and across states. We have established the result first obtained by Shavell and Weiss (1979) and restated by Hopenhayn and Nicolini (1997).

Proposition 2. *Over the spell of unemployment, benefits decrease, $b_t^* > b_{t+1}^*$. If the cost function is strictly convex, inequality* (4.30) *implies $V_t^{u*} > V_{t+1}^{u*}$. Consumption is not fully smoothed across states of employment and over time.*

Hopenhayn and Nicolini (1997, pp. 421, 435) then move on to demonstrate that the wage tax is not independent of the unemployment history. That means, as t varies, V_t^{e*} changes as well. We want to find out the the direction of this change. When we rearrange equations (4.22), (4.29) and the incentive-compatibility constraint (4.20), we arrive at the following expression[4]:

$$C(V_{t+1}^{u*}) - W(V_t^{e*}) = \frac{-p''(e_t^*)(1-p(e_t^*))p(e_t^*)}{[p'(e_t^*)]^3\beta}\big(W'(V_t^{e*}) - C'(V_{t+1}^{u*})\big) \tag{4.32}$$

$$= -\eta_t \frac{p''(e_t^*)}{(p'(e_t^*))^2\beta} \tag{4.33}$$

In the following, we work with the two expressions above in order to prove the following

Proposition 3. *If $C(\cdot)$ is convex and either (a) $\big[-p''(e_t^*)(1-p(e_t^*))p(e_t^*)\big]/[p'(e_t^*)]^3$ or (b) $-p''(e_t^*)/(p'(e_t^*))^2$ increases in e_t^*, then we must have that $V_t^{e*} > V_{t+1}^{e*}$. The agent's continuation value from employment decreases; hence, the tax levied on him increases the longer the spell of unemployment lasts.*

Proof. The proof is by contradiction. Suppose V_t^{e*} increases between two periods $\hat{t}$ and $\hat{t}+1$. From proposition 2, V_t^{u*} decreases over time and we have $V_{\hat{t}}^{u*} > V_{\hat{t}+1}^{u*} > V_{\hat{t}+2}^{u*}$. Therefore, it must be that $V_{\hat{t}+1}^{e*} - V_{\hat{t}+2}^{u*} > V_{\hat{t}}^{e*} - V_{\hat{t}+1}^{u*}$. The incentive-compatibility constraints in the two periods are:

$$\beta p'(e_{\hat{t}}^*)\big(V_{\hat{t}}^{e*} - V_{\hat{t}+1}^{u*}\big) = 1 = \beta p'(e_{\hat{t}+1}^*)\big(V_{\hat{t}+1}^{e*} - V_{\hat{t}+2}^{u*}\big) \tag{4.34}$$

This implies $p'(e_{\hat{t}}^*) > p'(e_{\hat{t}+1}^*)$ or equivalently $e_{\hat{t}}^* < e_{\hat{t}+1}^*$. The agent's search activity increases between the two periods.

[4] For the complete derivation, see appendix B.

Furthermore, we have $V_{\hat{t}}^{e*} < V_{\hat{t}+1}^{e*}$ which implies $W(V_{\hat{t}}^{e*}) < W(V_{\hat{t}+1}^{e*})$. Thus, $C(V_t^{u*}) - W(V_t^{e*})$ decreases between the two periods (the minuend decreases and the subtrahend increases) and $W'(V_t^{e*}) - C'(V_{t+1}^{u*})$ increases between the two periods (the minuend increases and the subtrahend decreases). If condition (a) holds between the two periods, the left hand side of equation (4.32) decreases, whereas the right hand side increases. We have a contradiction.

First-order conditions (4.25) and (4.26) can be written as:

$$W'(V_t^{e*}) - C'(V_t^{u*}) = \eta_t \frac{p'(e_t^*)}{p(e_t^*)} \tag{4.35}$$

Under the current assumptions, the left hand side increases between the two periods, whereas the ratio on the right hand side decreases. Hence, we must have $\eta_{\hat{t}+1} > \eta_{\hat{t}} > 0$. Then, under condition (b), the left hand side of equation (4.33) decreases, whereas the right hand side increases. We have a contradiction.

We conclude that under either condition (a) or (b), there cannot be two periods with V_t^{e*} increasing. Q.E.D.

So far we have shown that V_t^{u*} and V_t^{e*} (under some restrictions) decrease over time. We now want to study how these findings affect the search effort. To do this, totally differentiate the incentive-compatibility constraint (4.20) with respect to e_t^* and the incentive margin $(V_t^{e*} - V_{t+1}^{u*})$ and condition (b) reappears:

$$\frac{de_t^*}{d(V_t^{e*} - V_{t+1}^{u*})} = -\frac{1}{\frac{p''(e_t^*)}{\beta(p'(e_t^*))^2}} = -\frac{\beta(p'(e_t^*))^2}{p''(e_t^*)} \tag{4.36}$$

Given condition (b), search effort increases over time if the incentive margin increases. Equation (4.36) further shows that the effort response to increased incentives decreases as effort increases. Hence, a rising incentive margin reduces the increase in search effort. Getting one more unit of search effort becomes increasingly costly in terms of the incentive margin. In order to make e_t^* rise over the spell of unemployment, V_t^{e*} and V_{t+1}^{u*} may not fall arbitrarily. Look at first-order condition (4.22) that equates the principal's marginal benefit from search and the agent's marginal cost from search[5]. Given that e_t^* increases over time, $p'(e_t^*)$ falls and the second factor on the left hand side increases. On the right hand side, if $p''(e_t^*)$ increases significantly, it becomes less negative and marginal cost of search are reduced. Then it may be optimal to increase effort. In order to ensure this effect, V_t^{e*} has to fall by a smaller amount than V_{t+1}^{u*}. In this case, the incentive margin

[5]See also the discussion of the cost-minimizing effort level under perfect information and the incentive complications on page 20.

rises even if V_t^{e*} and V_{t+1}^{u*} fall over time.

Hopenhayn and Nicolini (1997) further demonstrate that, given condition (b), V_t^{u*} is an increasing function of the initially promised value of unemployment, V_0^u. From this, they deduce that all net transfers increase with V_0^u.

We can also graphically study the present environment. At the optimal point in period t, we must have:

$$\frac{u'(b_t^*)}{u'(w_t^*)} = 1 + \eta_t \frac{p'(e_t^*)}{p(e_t^*)} \tag{4.37}$$

Look at figure 2 on page 26-a. To begin, the agent's MRS can be derived analogously as in the case with perfect information on page 21. However, as above, I am not able to deduce the principal's MRT because I do not know how the incentive-compatibility constraint affects the principal's cost function and its shape. To derive its derivative at the point of optimality, I use the first-order conditions (4.23) and (4.25). At the optimal point, we must have MRS=MRT. Therefore, I first compute the MRS at the optimal and this gives the MRT. But this does not characterize the whole shape of the cost function isoquant. The MRS evaluated at the optimal triple (b_t^*, w_t^*, e_t^*) is given by:

$$\text{MRS} = -\frac{u'(b_t^*)}{u'(w_t^*)} \left(p(e_t^*) \frac{\beta}{1-\beta} \right)^{-1} = -\frac{1 + \eta_t \frac{p'(e_t^*)}{p(e_t^*)} u'(b_t^*)}{p(e_t^*) \frac{\beta}{1-\beta}} = \text{MRT} \tag{4.38}$$

We see that condition (4.37) is equivalent to condition (4.38). The optimal pair (b_t^*, w_t^*) lies above the 45-degree-line.

Unfortunately, it is not possible to depict the dynamics of the model. Look at the MRS. The denominator increases since e_t^* increases. The quotient of the marginal utilities of benefits and after-tax income can rise or fall depending on whether b_t^* falls faster than w_t^*. This leaves the slope at the optimal point in the next period undetermined. Look at the magnitude of the MRT at the optimal point as time unfolds. The denominator increases as e_t^* rises. In the numerator, the relative change in the probability falls and the marginal utility of benefits rises. From equation (4.35), η_t can fall, rise or stay constant depending on how V_t^e and V_t^u fall in relation to each other and how $p(e_t^*)$ changes. Thus, η_t's change is also undetermined. This leaves the numerator's behavior unknown. To sum up, from an analytical point of view, we cannot draw further conclusions about the model's dynamics. In particular, we do not know how the ratio b_t^*/w_t^* evolves over time.

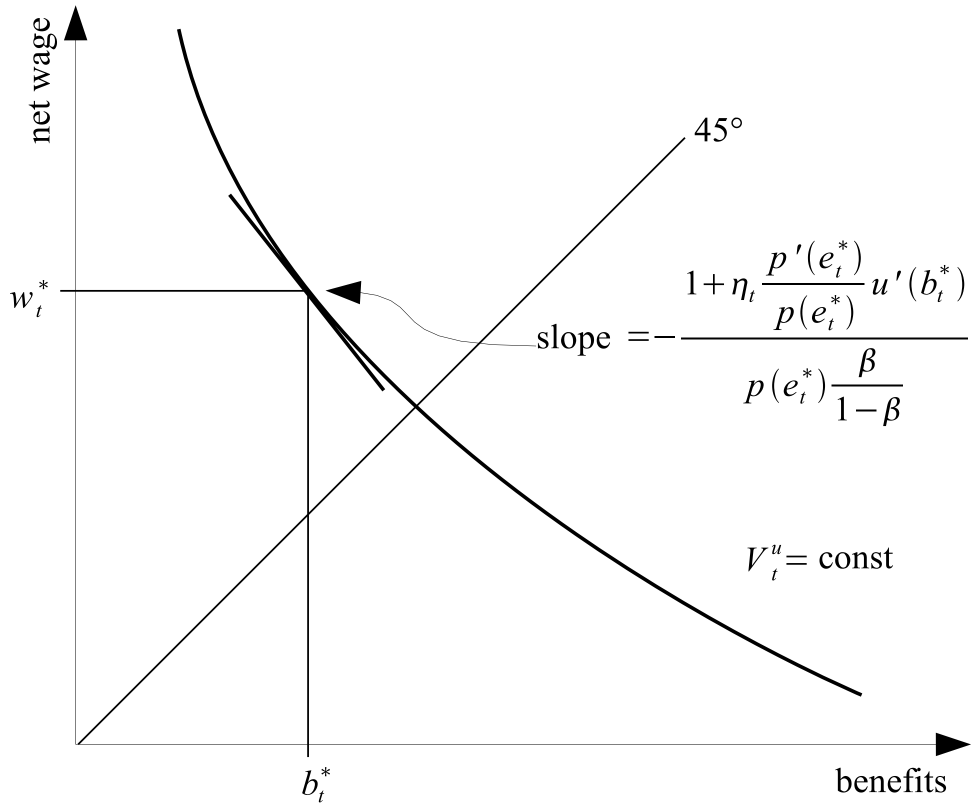

Figure 2: Illustrates the model without administrative costs under imperfect information.

5 Model with Administrative Costs

5.1 Introduction

This chapter now extends the preceding model by allowing for administrative costs. As I mentioned in the beginning, they can arise on the benefit distribution and on the tax collection side. If it is costly to distribute benefits, every time the principal pays out b_t, his cost functions rises by $a(b_t)$. If it is costly to collect taxes, the tax revenue $W(V_t^e)$ is reduced by $a(W(V_t^e))$.

The basic approach to study those effects is the same as the one described before. The only thing that changes is the principal's cost function (4.9). Whenever there is a b_t, we have to add $a(b_t)$ and when there is a $W(V_t^e)$, we have to add $a(W(V_t^e))$. I will now directly state the Bellman equation for a variant with only benefit distribution costs, with only tax collection costs, and the last variant features both costs structures. With benefit distribution costs, tax collection costs and both cost structures, the Bellman equations are:

$$C(V_t^u) = \min_{b_t, e_t, V_t^e, V_{t+1}^u} b_t + a(b_t) + \beta\big[p(e_t)W(V_t^e) + (1 - p(e_t))C(V_{t+1}^u)\big] \tag{5.1a}$$

$$C(V_t^u) = \min_{b_t, e_t, V_t^e, V_{t+1}^u} b_t + \beta\big[p(e_t)\big(W(V_t^e) + a(W(V_t^e))\big) + (1 - p(e_t))C(V_{t+1}^u)\big] \tag{5.1b}$$

$$C(V_t^u) = \min_{b_t, e_t, V_t^e, V_{t+1}^u} b_t + a(b_t) + \beta\big[p(e_t)\big(W(V_t^e) + a(W(V_t^e))\big) + (1 - p(e_t))C(V_{t+1}^u)\big] \tag{5.1c}$$

To distinguish the three variants, I will label variables and parameters with Roman uppercase numbers. $\iota = I, II, III$ refers to the model with benefit distribution costs, tax collection costs and both cost structures. ι alone is used if the expression holds for all cost structures I, II and III. Figures without a Roman number refer to the case without administrative costs.

In the next two sections, I will first analyze the principal's problem under perfect information and then under imperfect information. The analysis of the three variants will be done simultaneously because, as it turns out, the qualitative results are the same.

5.2 Perfect Information

The principal's problem in period t is to solve problems (5.1) each subject to the promise-keeping constraint (4.10) and initial condition (4.11). Stating the Lagrangian for each variant is straightforward. I will simply give the first-order conditions and the corresponding envelope conditions. With benefit distribution costs, tax collection costs and both cost structures, we have:

$$\lambda_t^I = \frac{1 + a'(b_t^{I*})}{u'(b_t^{I*})} = W'(V_t^{eI*}) = \frac{1}{u'(w_t^{I*})} = C'(V_{t+1}^{uI*}) = C'(V_t^{uI*}) \tag{5.2a}$$

$$\begin{aligned} \lambda_t^{II} &= \frac{1}{u'(b_t^{II*})} = W'(V_t^{eII*})\big(1 + a'(W(V_t^{eII*}))\big) \\ &= \frac{1 + a'(W(V_t^{eII*})}{u'(w_t^{II*})} = C'(V_{t+1}^{uII*}) = C'(V_t^{uII*}) \end{aligned} \tag{5.2b}$$

$$\begin{aligned} \lambda_t^{III} &= \frac{1 + a'(b_t^{III*})}{u'(b_t^{III*})} = W'(V_t^{eIII*})\big(1 + a'(W(V_t^{eIII*}))\big) \\ &= \frac{1 + a'(W(V_t^{eIII*})}{u'(w_t^{III*})} = C'(V_{t+1}^{uIII*}) = C'(V_t^{uIII*}) \end{aligned} \tag{5.2c}$$

All three conditions imply that benefits and wage income stay constant over time. The latter implies that the the continuation value of employment stays constant. Hence, we have $b_t^{\iota*} = b_{t+1}^{\iota*}$ and $w_t^{\iota*} = w_{t+1}^{\iota*}$. If unemployed or employed, the agent's consumption is fully smoothed over time. If the cost function is convex, the continuation value of unemployment is constant over time, $V_t^{u\iota*} = V_{t+1}^{u\iota*}$.

In the three environments, it is best for the principal to prescribe a constant level of search effort, $e_t^{\iota} = e_{t+1}^{\iota}$ in all periods, for the same reasons as explained on page 19 when we studied the environment under perfect information without administrative costs.

From now on we drop the time index t and work with $w_t^{\iota} = \overline{w}^{\iota}$, $b_t^{\iota} = b^{\iota}$, $e_t^{\iota} = e^{\iota}$, $\lambda_t^{\iota} = \lambda^{\iota}$ and $V_t^{u\iota} = V^{u\iota}$.

Conditions (5.2) further imply:

$$\frac{u'(b^{I*})}{u(\overline{w}^{I*})} = 1 + a'(b^{I*}) \tag{5.3a}$$

$$\frac{u'(b^{II*})}{u(\overline{w}^{II*})} = \frac{1}{1 + W'(V_t^{eII*})} \tag{5.3b}$$

$$\frac{u'(b^{III*})}{u(\overline{w}^{III*})} = \frac{1 + a'(b^{III*})}{1 + W'(V_t^{eIII*})} \tag{5.3c}$$

Conditions (5.3) imply $b^{\iota*} < \overline{w}^{\iota*}$. Thus, consumption is not fully smoothed across states, contrary to the result obtained above where we have $b^{\iota*} = \overline{w}^{\iota*}$. The reason for this is that the UI is not actuarially fair anymore. The full amount of expected discounted taxes is not paid out in form of benefits because taxes also have to cover administrative costs which can be deemed as transaction costs. These costs bring about the no-full-insurance result even though there are no informational asymmetries.

It is left to calculate the cost-minimizing search effort. We can use this and our previous findings to give an expression to the cost function:

$$\begin{aligned}
C(V^{uI}) &= \frac{\overline{w}^{I*}}{1-\beta} - \frac{\beta p(e^I)w}{(1-\beta)(1-\beta(1-p(e^I)))} + \frac{a(b^{I*}) - \tau^I \overline{w}^{I*}}{1-\beta(1-p(e^I))} \\
C(V^{uII}) &= \frac{\overline{w}^{II*}}{1-\beta} - \frac{\beta p(e^{II})w}{(1-\beta)\big(1-\beta(1-p(e^{II}))\big)} + \frac{\beta p(e^{II})a(W(V_t^{eII*})) - \tau^{II}\overline{w}^{II*}}{1-\beta(1-p(e^{II}))} \\
C(V^{uIII}) &= \frac{\overline{w}^{III*}}{1-\beta} - \frac{\beta p(e^{III})w}{(1-\beta)\big(1-\beta(1-p(e^{III}))\big)} \\
&\quad + \frac{a(b^{III*}) + \beta p(e^{III})a(W(V^{eIII*})) - \tau^{III}\overline{w}^{III*}}{1-\beta(1-p(e^{III}))}
\end{aligned}$$

where I have set $b^{\iota*} = (1-\tau^\iota)\overline{w}^{\iota*}$, because both figures stay constant over time. We can also rearrange the promise-keeping constraint:

$$V^{u\iota} = \frac{u(b^{\iota*}) - e^\iota + \frac{\beta}{1-\beta}u(\overline{w}^{\iota*})}{1-\beta(1-p(e^\iota))} \tag{5.4}$$

The Lagrangian functions for choosing a cost minimizing $e^{\iota*}$ are:

$$\begin{aligned}
\mathcal{L}^I &= \frac{1}{u'(\overline{w}^{I*})}\Bigg[-\frac{\overline{w}^{I*}u'(\overline{w}^{I*})}{1-\beta} + \frac{\beta p(e^I)u'(\overline{w}^{I*})w}{(1-\beta)(1-\beta(1-p(e^I)))} - u'(\overline{w}^{I*})\frac{a(b^{I*}) - \tau^I\overline{w}^{I*}}{1-\beta(1-p(e^I))} \\
&\quad + \frac{u(b^{I*}) - e^I + \frac{\beta}{1-\beta}u(\overline{w}^{I*})}{1-\beta(1-p(e^I))}\Bigg] \\
\mathcal{L}^{II} &= \frac{1}{u'(\overline{w}^{II*})}\Bigg[-\frac{\overline{w}^{II*}u'(\overline{w}^{II*})}{1-\beta} + \frac{\beta p(e^{II})u'(\overline{w}^{II*})w}{(1-\beta)(1-\beta(1-p(e^{II})))} \\
&\quad - u'(\overline{w}^{II*})\frac{a(W(V^{eII*})) - \tau^{II}\overline{w}^{II*}}{1-\beta(1-p(e^{II}))} + \frac{u(b^{II*}) - e^{II} + \frac{\beta}{1-\beta}u(\overline{w}^{II*})}{1-\beta(1-p(e^{II}))}\Bigg] \\
\mathcal{L}^{III} &= \frac{1}{u'(\overline{w}^{III*})}\Bigg[-\frac{\overline{w}^{III*}u'(\overline{w}^{III*})}{1-\beta} + \frac{\beta p(e^{III})u'(\overline{w}^{III*})w}{(1-\beta)(1-\beta(1-p(e^{III})))} \\
&\quad - u'(\overline{w}^{III*})\frac{a(b^{III*}) + a(W(V^{eIII*})) - \tau^{III}\overline{w}^{III*}}{1-\beta(1-p(e^{III}))} \\
&\quad + \frac{u(b^{III*}) - e^{III} + \frac{\beta}{1-\beta}u(\overline{w}^{III*})}{1-\beta(1-p(e^{III}))}\Bigg]
\end{aligned}$$

$e^{\iota *}$ maximizes the following expressions:

$$\begin{aligned} e^{I*} &= \arg\max_{e^I} \sum_{t=0}^{\infty} \big(\beta(1-p(e^I))\big)^t \left(p(e^I)\frac{\beta}{1-\beta}u'(\overline{w}^{I*})w - e^I\right) \\ &\quad + \arg\max_{e^I} \sum_{t=0}^{\infty} \big(\beta(1-p(e^I))\big)^t \left(-a(b^{I*}) + \tau^I \overline{w}^{I*} + u(b^{I*}) + \frac{\beta}{1-\beta}u(\overline{w}^{I*})\right) \end{aligned} \tag{5.5a}$$

$$\begin{aligned} e^{II*} &= \arg\max_{e^{II}} \sum_{t=0}^{\infty} \big(\beta(1-p(e^{II}))\big)^t \left(p(e^{II})\frac{\beta}{1-\beta}u'(\overline{w}^{II*})w - e^{II}\right) \\ &\quad + \arg\max_{e^{II}} \sum_{t=0}^{\infty} \big(\beta(1-p(e^{II}))\big)^t \Big(-a(W(V^{eII*})) + \tau^{II}\overline{w}^{II*} \\ &\qquad + u(b^{II}) + \frac{\beta}{1-\beta}u(\overline{w}^{II*})\Big) \end{aligned} \tag{5.5b}$$

$$\begin{aligned} e^{III*} &= \arg\max_{e^{III}} \sum_{t=0}^{\infty} \big(\beta(1-p(e^{III}))\big)^t \left(p(e^{III})\frac{\beta}{1-\beta}u'(\overline{w}^{III*})w - e^{III}\right) \\ &\quad + \arg\max_{e^{III}} \sum_{t=0}^{\infty} \big(\beta(1-p(e^{III}))\big)^t \Big(-a(b^{III*}) - a(W(V^{IIIe*})) + \tau^{III}\overline{w}^{III*} \\ &\qquad + u(b^{III*}) + \frac{\beta}{1-\beta}u(\overline{w}^{III*})\Big) \end{aligned} \tag{5.5c}$$

The first part of expressions (5.5) is similar to expression (4.15). Note, however, that after tax consumption is different, $\overline{w}^* \neq \overline{w}^{\iota *}$. Hence, on first sight, we cannot tell whether the first part of expressions (5.5) is bigger or smaller than expression (4.15).

Consider the second part of expressions (5.5) and rewrite $-a(b^{I*}) + \tau^I \overline{w}^{I*} = \overline{w}^{I*} + \big(a(b^{I*}) - b^{I*}\big)$ and $-a(W(V^{ie*})) + \tau^i \overline{w}^{i*} = \overline{w}^{i*} + \big(a(W(V^{ie*})) - b^{i*}\big)$ for $i = II, III$. If we assume that benefit distribution costs are not larger than benefits, $a(b^{I*}) < b^{I*}$, tax collection costs are not larger than benefits, $a(W(V^{IIe*})) < b^{II*}$ and benefit distribution plus tax collection costs are not larger than benefits, $a(b^{III*}) + a(W(V^{IIIe*})) < b^{III*}$, the sign of the last expressions of (5.5) is undetermined. Therefore, we cannot compare e^* and $e^{\iota *}$. Obviously, we also cannot compare $e^{\iota *}$ to e^{j*} for $j = I, II, III$, $j \neq \iota$.

We can further study optimality conditions (5.3). Similarly to the model without administration costs, we can derive the agent's MRS and the principal's MRT:

$$\text{MRS} := \frac{d\overline{w}^{\iota}}{db^{\iota}} = -\frac{u'(b^{\iota})}{u'(\overline{w}^{\iota})}\left(p(e^{\iota})\frac{\beta}{1-\beta}\right)^{-1} \tag{5.6a}$$

$$\text{MRT} := \frac{d\overline{w}^{I}}{db^{I}} = -\big(1 + a'(b^I)\big)\left(p(e^{I})\frac{\beta}{1-\beta}\right)^{-1} \tag{5.6b}$$

$$\text{MRT} := \frac{d\overline{w}^{II}}{db^{II}} = -\frac{1}{1 + a'(W(V^{eII}))}\left(p(e^{II})\frac{\beta}{1-\beta}\right)^{-1} \tag{5.6c}$$

$$\text{MRT} := \frac{d\overline{w}^{III}}{db^{III}} = -\frac{1 + a'(b^{III})}{1 + a'(W(V^{eIII}))}\left(p(e^{III})\frac{\beta}{1-\beta}\right)^{-1} \tag{5.6d}$$

Conditions (5.3) are equivalent to the familiar condition MRS = MRT that must hold at the optimal point. The environment with benefit distribution costs is depicted in figure 3, page 31-a. Qualitatively, figures for the two other cases are the same. Moreover, we cannot include all three cases in one picture because we do not precisely know what the optimal triple $(b^{\iota *}, \overline{w}^{\iota *}, e^{\iota *})$ in each variant is. The curve of transformation's curvature depends on $p(e^I)$, β and benefit distribution costs $a(b^I)$. Because of the benefit distribution costs, the line of transformation is actually a curve of transformation. Its concavity shows that financing one more unit of benefits becomes increasingly costly in terms of foregone after-tax income. Especially, when it approaches zero, marginal costs of providing more benefits are infinite. The only means to rise benefits then is to spend more resources and the curve of transformation shifts to the right.

From conditions (5.3b) and (5.3c), we can further draw conclusions regarding the tax revenue. Since the left hand side is positive and for $W(V_t^{ei*}) < 0$ we have $a'(W(V_t^{ei*})) < 0$ for $i = II, III$. The following proposition must hold:

Proposition 4. *Collecting taxes $W(V_t^{ei*})$ must satisfy $-1 < a'(W(V_t^{ei*})) < 0$ for $i = II, III$.*

That means $W(V_t^{ei*})$ may not fall below a point with $a'(W(V_t^{ei*})) \leq -1$. The derivative $a'(W(V_t^{ei*}))$ denotes the marginal cost of collecting taxes. If tax revenue is large, the derivative is approximately equal to the incremental cost $a(W(V_t^{ei*})) - a(W(V_t^{ei*}) - 1) < 0$, that is, the cost of collecting one more unit of taxes. Hence, if we had $a'(W(V_t^{ei*})) \leq -1$, then $a(W(V_t^{ei*})) - a(W(V_t^{ei*}) - 1) \leq -1$ which is equivalent to $a(W(V_t^{ei*})) + 1 \leq a(W(V_t^{ei*}) - 1)$. Costs of collecting one more unit of taxes $a(W(V_t^{ei*}) - 1)$ are greater than or equal to tax collection costs plus the additional unit of tax revenue. Hence, each additional unit of taxes would be eaten up by tax collection costs and nothing would be left for benefits. Hence, proposition 4 gives an upper bound of the size of the tax revenue.

5.3 Imperfect Information

The section proceeds analogously to the study of Hopenhayn and Nicolini's model. We again study the three cases of administrative costs simultaneously. The cost functions in period t in the three cases are given by (5.1). The promise-keeping constraint that

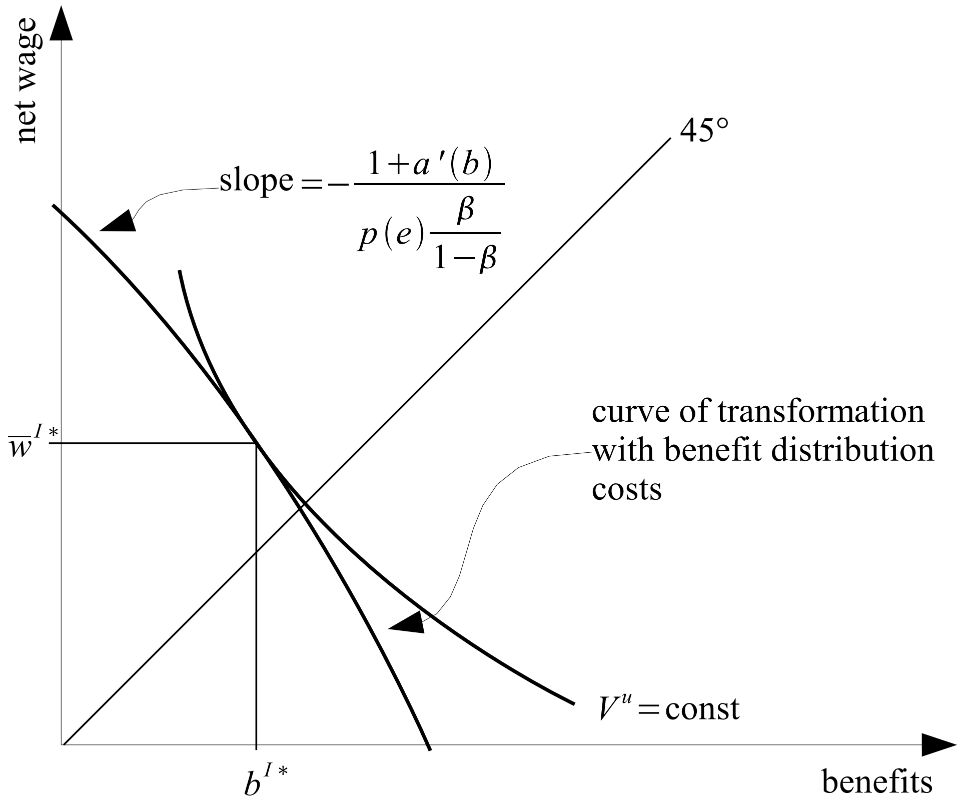

Figure 3: Illustrates the model with benefit distribution costs under perfect information.

has to hold in period t is defined by (4.10). Now an additional constraint enters the principal's cost-minimization problem. It is the incentive-compatibility constraint given by (3.2). In the following three subsections, I will only state the sufficient and necessary first-order conditions along with an analysis of them. The Lagrange multiplier on the incentive-compatibility constraint in period t will be η_t^ι.

With benefit distribution costs, tax collection costs and benefit distribution and tax collection costs the first-order conditions with respect to b_t^ι, $V_t^{e\iota}$, $V_{t+1}^{u\iota}$ and the envelope conditions are:

$$\lambda_t^I = \frac{1+a'(b_t^{I*})}{u'(b_t^{I*})} = W'(V_t^{eI*}) - \eta_t^I \frac{p'(e_t^{I*})}{p(e_t^{I*})} = C'(V_{t+1}^{uI*}) + \eta_t^I \frac{p'(e_t^{I*})}{1-p(e_t^{I*})} \tag{5.7a}$$

$$\begin{aligned}\lambda_t^{II} &= \frac{1+a'(b_t^{II*})}{u'(b_t^{II*})} = W'(V_t^{eII*})\big(1+a'(W(V_t^{eII*}))\big) - \eta_t^{II} \frac{p'(e_t^{II*})}{p(e_t^{II*})} \\ &= C'(V_{t+1}^{uII*}) + \eta_t^{II} \frac{p'(e_t^{II*})}{1-p(e_t^{II*})}\end{aligned} \tag{5.7b}$$

$$\begin{aligned}\lambda_t^{III} &= \frac{1+a'(b_t^{III*})}{u'(b_t^{III*})} = W'(V_t^{eIII*})\big(1+a'(W(V_t^{eIII*}))\big) - \eta_t^{III*} \frac{p'(e_t^{III*})}{p(e_t^{III*})} \\ &= C'(V_{t+1}^{uIII*}) + \eta_t^{III} \frac{p'(e_t^{III*})}{1-p(e_t^{III*})}\end{aligned} \tag{5.7c}$$

The first-order conditions with respect to e_t^ι are:

$$p'(e_t^{I*})\big(W(V_t^{eI*}) - C(V_{t+1}^{uI*})\big) = \eta_t^I p''(e_t^{I*})\big(V_t^{eI*} - V_{t+1}^{uI*}\big) \tag{5.8a}$$

$$p'(e_t^{II*})\big(W(V_t^{eII*}) + a(W(V_t^{eII*})) - C(V_{t+1}^{uII*})\big) = \eta_t^{II} p''(e_t^{II*})\big(V_t^{eII*} - V_{t+1}^{uII*}\big) \tag{5.8b}$$

$$p'(e_t^{III*})\big(W(V_t^{eIII*}) + a(W(V_t^{eIII*}) - C(V_{t+1}^{uIII*})\big) = \eta_t^{III} p''(e_t^{III*})\big(V_t^{eIII*} - V_{t+1}^{uIII*}\big) \tag{5.8c}$$

The envelope conditions can be written as:

$$C'(V_t^{uI*}) = \lambda_t^I = \frac{1+a'(b_t^{I*})}{u'(b_t^{I*})} = \big(1-p(e_t^{I*})\big)C'(V_{t+1}^{uI*}) + p(e_t^{I*})W'(V_t^{eI*}) \tag{5.9a}$$

$$\begin{aligned}C'(V_t^{uII*}) = \lambda_t^{II} &= \frac{1}{u'(b_t^{II*})} \\ &= \big(1-p(e_t^{II*})\big)C'(V_{t+1}^{uIII*}) + p(e_t^{II*})W'(V_t^{eII*})\big(1+a'(W(V_t^{eII*}))\big)\end{aligned} \tag{5.9b}$$

$$\begin{aligned}C'(V_t^{uIII*}) = \lambda_t^{III} &= \frac{1+a'(b_t^{III*})}{u'(b_t^{III*})} \\ &= \big(1-p(e_t^{III*})\big)C'(V_{t+1}^{uIII*}) + p(e_t^{III*})W'(V_t^{eIII*})\big(1+a'(W(V_t^{eIII*}))\big)\end{aligned} \tag{5.9c}$$

The first-order conditions (5.7) can be rearranged to produce:

$$W'(V_t^{eI*}) - C'(V_{t+1}^{uI*}) = \eta_t^I p'(e_t^{I*}) \left(\frac{1}{1-p(e_t^{I*})} + \frac{1}{p(e_t^{I*})} \right) \tag{5.10a}$$

$$W'(V_t^{eII*})\big(1 + a'(W(V_t^{eII*}))\big) - C'(V_{t+1}^{uII*}) = \eta_t^{II} p'(e_t^{II*}) \left(\frac{1}{1-p(e_t^{II*})} + \frac{1}{p(e_t^{II*})} \right) \tag{5.10b}$$

$$W'(V_t^{eIII*})\big(1 + a'(W(V_t^{eIII*}))\big) - C'(V_{t+1}^{uIII*}) = \eta_t^{III} p'(e_t^{III*}) \left(\frac{1}{1-p(e_t^{III*})} + \frac{1}{p(e_t^{III*})} \right) \tag{5.10c}$$

How continuation values of unemployment and employment evolve over time depends on the sign of η_t^{ι}. In the environment without administrative costs, Hopenhayn and Nicolini (1997) were able to prove the lemma that $\eta_t > 0$. In the current environment, however, I am not able to verify this result. I will show this in the following.

The analysis closely follows Hopenhayn and Nicolini's proof of the lemma. If $\eta_t^{\iota} > 0$, equations (5.9) and (5.10) give:

$$W'(V_t^{eI*}) > C'(V_t^{uI*}) > C'(V_{t+1}^{uI*}) \tag{5.11a}$$

$$W'(V_t^{eII*})\big(1 + a'(W(V_t^{eII*}))\big) > C'(V_t^{uII*}) > C'(V_{t+1}^{uII*}) \tag{5.11b}$$

$$W'(V_t^{eIII*})\big(1 + a'(W(V_t^{eIII*}))\big) > C'(V_t^{uIII*}) > C'(V_{t+1}^{uIII*}) \tag{5.11c}$$

To establish $\eta_t^{\iota} > 0$, we suppose the contrary, $\eta_t^{\iota} \leq 0$. Then we have:

$$W'(V_t^{eI*}) \leq C'(V_t^{uI*}) \leq C'(V_{t+1}^{uI*}) \tag{5.12a}$$

$$W'(V_t^{eII*})\big(1 + a'(W(V_t^{eII*}))\big) \leq C'(V_t^{uII*}) \leq C'(V_{t+1}^{uII*}) \tag{5.12b}$$

$$W'(V_t^{eIII*})\big(1 + a'(W(V_t^{eIII*}))\big) \leq C'(V_t^{uIII*}) \leq C'(V_{t+1}^{uIII*}) \tag{5.12c}$$

Using the incentive-compatibility constraint (3.2) relations (5.12) imply:

$$V_t^{e\iota*} > V_{t+1}^{u\iota*} \geq V_t^{u\iota*} \tag{5.13}$$

Furthermore, equations (5.12) imply:

$$\frac{1 + a'(b_t^{I*})}{u'(b_t^{I*})} \geq \frac{1}{u'(w_t^{I*})} \quad \Leftrightarrow \quad 1 + a'(b_t^{I*}) \geq \frac{u'(b_t^{I*})}{u'(w_t^{I*})}$$

$$\frac{1}{u'(b_t^{II*})} \geq \frac{1 + a'(W(V_t^{eII*}))}{u'(w_t^{II*})} \quad \Leftrightarrow \quad \frac{1}{1 + a'(W(V_t^{eII*}))} \geq \frac{u'(b_t^{II*})}{u'(w_t^{II*})}$$

$$\frac{1+a'(b_t^{III*})}{u'(b_t^{III*})} \geq \frac{1+a'(W(V_t^{eIII*}))}{u'(w_t^{III*})} \Leftrightarrow \frac{1+a'(b_t^{III*})}{1+a'(W(V_t^{eIII*}))} \geq \frac{u'(b_t^{III*})}{u'(w_t^{III*})}$$

In each case, two possibilities can arise:

$$\frac{u'(b_t^{I*})}{u'(w_t^{I*})} \leq 1 \leq 1+a'(b_t^{I*}) \qquad \Rightarrow b_t^{I*} \geq w_t^{I*} \tag{5.14a}$$

$$1 \leq \frac{u'(b_t^{I*})}{u'(w_t^{I*})} \leq 1+a'(b_t^{I*}) \qquad \Rightarrow b_t^{I*} \leq w_t^{I*} \tag{5.14b}$$

$$\frac{u'(b_t^{II*})}{u'(w_t^{II*})} \leq 1 \leq \frac{1}{1+a'(W(V_t^{eII*}))} \qquad \Rightarrow b_t^{II*} \geq w_t^{II*} \tag{5.14c}$$

$$\frac{1}{1+a'(W(V_t^{eII*}))} \leq \frac{u'(b_t^{II*})}{u'(w_t^{II*})} \leq 1+a'(b_t^{II*}) \qquad \Rightarrow b_t^{II*} \leq w_t^{II*} \tag{5.14d}$$

$$\frac{u'(b_t^{III*})}{u'(w_t^{III*})} \leq 1 \leq \frac{1+a'(b_t^{III*})}{1+a'(W(V_t^{eIII*}))} \qquad \Rightarrow b_t^{III*} \geq w_t^{III*} \tag{5.14e}$$

$$1 \leq \frac{u'(b_t^{III*})}{u'(w_t^{III*})} \leq \frac{1+a'(b_t^{III*})}{1+a'(W(V_t^{eIII*}))} \qquad \Rightarrow b_t^{III*} \leq w_t^{III*} \tag{5.14f}$$

We can rewrite the the promise-keeping constraint (4.10) and obtain[6]:

$$\underbrace{u(b_t^{\iota*}) - u(w_t^{\iota*})}_{\#1} + \underbrace{\beta p(e_t^{\iota*})\big(V_t^{e\iota*} - V_{t+1}^{u\iota*}\big) - e_t^{\iota*}}_{\#2} = (\beta-1)\big(V_t^{e\iota*} - V_{t+1}^{u\iota*}\big) - \big(V_{t+1}^{u\iota*} - V_t^{u\iota*}\big) \tag{5.15}$$

Consider the first possibility in the three cases. By (5.14a), (5.14c) and (5.14e), item #1 is nonnegative. Item #2 displays that part of the agent's expected utility function he maximizes over the choice of e_t^{ι} and that then produces the incentive-compatibility constraint. Any feasible $e_t^{\iota} \geq 0$ cannot make this term less than zero. Hence, item #2 is nonnegative. In total, the left hand side is nonnegative. By relation (5.13), the right hand side is negative. This yields a contradiction and we are forced to rule out possibilities (5.14a), (5.14c) and (5.14e).

Now consider possibilities (5.14b), (5.14d) and (5.14f). They do not yield a contradiction when we study equation (5.15). Therefore, we cannot rule out the second possibility and $\eta_t^{\iota} \leq 0$.

If we redo the analysis above in order to establish $\eta_t^{\iota} < 0$ and suppose the contrary, $\eta_t^{\iota} \geq 0$, we get:

$$W'(V_t^{eI*}) \geq C'(V_t^{uI*}) \geq C'(V_{t+1}^{uI*}) \tag{5.16a}$$

$$W'(V_t^{eII*})\big(1+a'(W(V_t^{eII*}))\big) \geq C'(V_t^{uII*}) \geq C'(V_{t+1}^{uII*}) \tag{5.16b}$$

[6] For the manipulations, see appendix C

$$W'(V_t^{eIII*})\big(1+a'(W(V_t^{eIII*}))\big) \geq C'(V_t^{uIII*}) \geq C'(V_{t+1}^{uIII*}) \tag{5.16c}$$

The last inequalities and the incentive-compatibility constraint (4.20) imply

$$V_t^{u\iota *} \geq V_{t+1}^{u\iota *} \tag{5.17}$$

$$V_t^{e\iota *} > V_{t+1}^{u\iota *} \tag{5.18}$$

Unlike in the case without administrative costs, we cannot deduce a relationship between $V_t^{e\iota *}$ and $V_{t+1}^{u\iota *}$.

Relations (5.16) imply a third possibility:

$$\frac{1}{u'(w_t^{I*})} \geq \frac{1+a'(b_t^{I*})}{u'(b_t^{I*})} \qquad \Rightarrow b_t^{I*} \leq w_t^{I*}$$
$$\frac{1+a'(W(V_t^{eII*}))}{u'(w_t^{II*})} \geq \frac{1}{u'(b_t^{II*})} \qquad \Rightarrow b_t^{II*} \leq w_t^{II*}$$
$$\frac{1+a'(W(V_t^{eIII*}))}{u'(w_t^{III*})} \geq \frac{1+a'(b_t^{III*})}{u'(b_t^{III*})} \qquad \Rightarrow b_t^{III*} \leq w_t^{III*}$$

If we now study the rearranged promise-keeping constraint (5.15), we see that item #1 is nonpositive and #2 is nonnegative. This leaves the left hand side's sign undetermined. By relations (5.17) and (5.18), the right hand side's sign is undetermined as well. Therefore, we cannot rule out the third possibility and $\eta_t^\iota \geq 0$.

To sum up the results obtained so far, we have two scenarios. In the first, I wanted to establish $\eta_t^\iota > 0$, supposed the contrary, $\eta_t^\iota \leq 0$, and was not able to rule out the following possibilities:

$$1 \leq \frac{u'(b_t^{I*})}{u'(w_t^{I*})} \leq 1 + a'(b_t^{I*}) \tag{5.19a}$$

$$1 < \frac{u'(b_t^{II*})}{u'(w_t^{II*})} < \frac{1}{1+a'(W(V_t^{eII*}))} \tag{5.19b}$$

$$1 < \frac{u'(b_t^{III*})}{u'(w_t^{III*})} < \frac{1+a'(b_t^{III*})}{1+a'(W(V_t^{eIII*}))} \tag{5.19c}$$

In the second, I wanted to establish $\eta_t^\iota < 0$, supposed the contrary, $\eta_t^\iota \geq 0$, and was not able to rule out the following possibility:

$$1 \leq 1 + a'(b_t^{I*}) \leq \frac{u'(b_t^{I*})}{u'(w_t^{I*})} \tag{5.20a}$$

$$1 < \frac{1}{1+a'(W(V_t^{eII*}))} < \frac{u'(b_t^{II*})}{u'(w_t^{II*})} \tag{5.20b}$$

$$1 < \frac{1 + a'(b_t^{III*})}{1 + a'(W(V_t^{eIII*}))} < \frac{u'(b_t^{III*})}{u'(w_t^{III*})} \tag{5.20c}$$

In both cases of the three variants, however, we have $w_t^{\iota *} \geq b_t^{\iota *}$. From the first-order conditions given by (5.7), we obtain $b_t^{\iota *} \geq b_{t+1}^{\iota *}$ ($b_t^{\iota *} \leq b_{t+1}^{\iota *}$) if and only if $\eta_t^{\iota} \geq 0$ ($\eta_t^{\iota} \leq 0$). Hence, in scenarios (5.19) the sequence of benefits can rise without bound. This hardly can be the characteristic of a cost-minimizing scheme. It is in sharp contrast to scenarios (5.20) where benefits can fall over time. Total costs would be higher in scenarios (5.19) than in scenarios (5.20). Therefore, scenarios (5.19) and $\eta_t^{\iota} \leq 0$ lack intuition, however, I am not able to provide further formal justifications.

In the sequel, we will proceed under the following

Assumption 1. *The Lagrange multiplier on the incentive-compatibility constraint is positive,* $\eta_t^{\iota} > 0$

The assumption implies relations (5.11). From them, we conclude $w_t^{\iota *} > b_t^{\iota *} > b_{t+1}^{\iota *}$. This verifies proposition 2.

Proposition 5. *With benefit collection costs, tax collection costs and both cost structures, proposition 2 holds under assumption 1.*

To prove that the continuation value of employment declines as well, we can rearrange equations (5.7) and (5.10) and the incentive-compatibility constraint (4.20) to produce equations (4.32) and (4.33). Regarding conditions (a) and (b), we note that assumption 1 has to hold so that the proof of proposition 3 works.

Proposition 6. *With benefit collection costs, tax collection costs and both cost structures, proposition 3 holds under assumption 1.*

Regarding the amount of taxes collected, we can draw further conclusions from using assumption 1 and inspecting relations (5.11b) and (5.11c) as in the case with perfect information:

Proposition 7. *Given assumption 1, we must have* $-1 < a'(W(V_t^{ei*})) < 0$, $i = II, III$.

The same interpretation holds as above on page 31.

We can study the present setting analogously to the case without administrative costs under imperfect information. I only consider benefit distribution costs and leave presenting the two remaining cases. They are qualitatively seen the same as the case with benefit distribution costs.The agent's MRS is defined by equation (4.17). Because of the incentive-compatibility constraint, I am not able to fully characterize the shape of a cost function isoquant. By use of the first-order conditions (5.7) and MRS=MRT, the following equation must hold at the optimal point:

$$\text{MRT}^I := -\frac{1 + a'(b_t^{I*}) + \eta_t^I \frac{p'(e_t^{I*})}{p(e_t^{I*})} u'(b_t^{I*})}{p(e_t^{I*}) \frac{\beta}{1-\beta}} = -\frac{u'(b_t^{I*})}{u'(w_t^{I*})} \left(p(e_t^{I*}) \frac{\beta}{1-\beta} \right)^{-1} =: \text{MRS}^I$$

The shape of the agent's indifference curve is fully characterized. The derivative of the principal's cost function isoquant is only known at the optimal point. The point of tangency must be to the left of the 45-degree line because of proposition 5. Figure 4 on page 37-a reports the environment with benefit distribution costs.

The optimal point lies to the left of the 45-degree line. Only the slopes are different because the analysis so far also does not tell us what the precise values of e_t^{I*}, e_t^{II*} and e_t^{III*} are. That is why we cannot analytically compare the three cases to each other and to the environment without administrative costs.

Using the graphical exposition developed above, we can draw assumption 1. At the optimal point, we must have:

$$\text{MRS}^I < -\frac{1 + a'(b_t^{I*})}{p(e_t^{I*}) \frac{\beta}{1-\beta}} \tag{5.21}$$

Figure 5 on page 37-b shows inequality (5.21). The slope of the line induced by assumption 1 is larger than the MRS at the optimal point. The difference between the slopes, which is positive due to the assumption, is the moral hazard factor. The MRT with the moral hazard factor shows that it is more costly to get one more unit of b_t^I in terms of foregone w_t^I than without moral hazard.

The preceding argument also shows why assuming $\eta_t^\iota < 0$ does not make sense. It would imply that the inequality sign of relation (5.21) would be reversed. Even though moral hazard would be present, it would be less costly to finance one more unit of benefits in terms of forgone wage income than in the case without moral hazard.

At the beginning of the graphical analysis, I explained why I am not sure about the precise line of transformation's shape. I only considered its derivative at the optimal point. The question now is whether relation (5.21) implies that the line of transformation's slope under imperfect information is steeper throughout than in the case with perfect information. I would not say so because we still do not know how search effort develops under imperfect information. This also crucially determines the slopes in question.

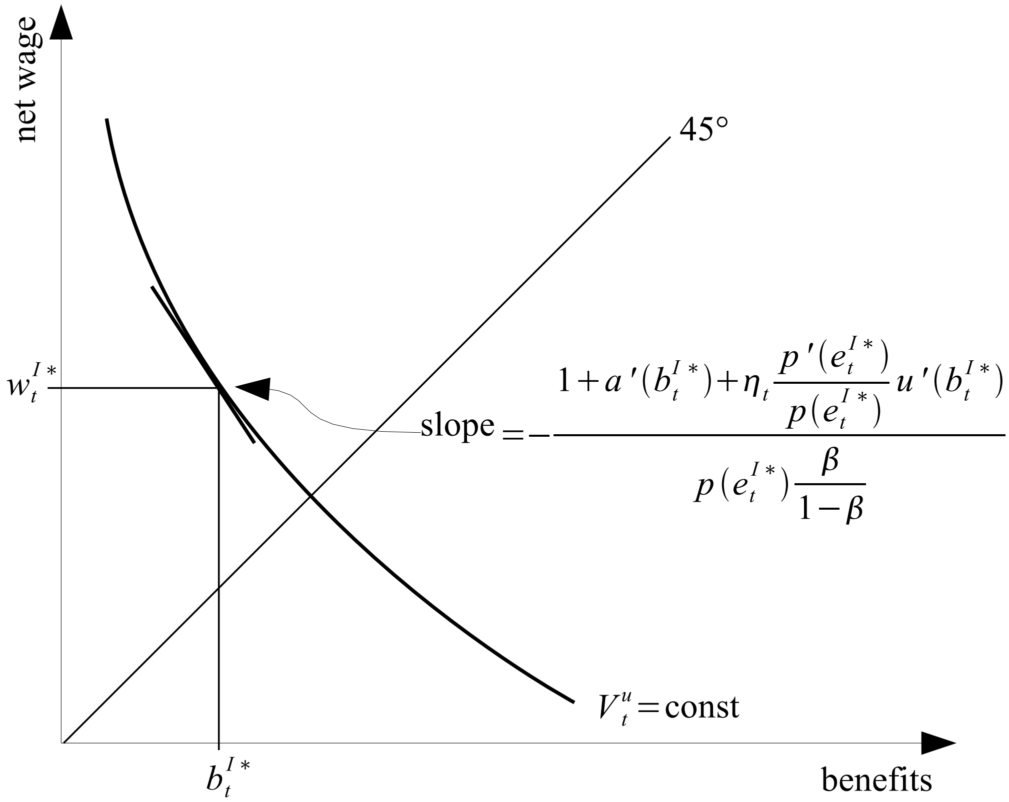

Figure 4: Illustrates the model with benefit distribution costs under imperfect information.

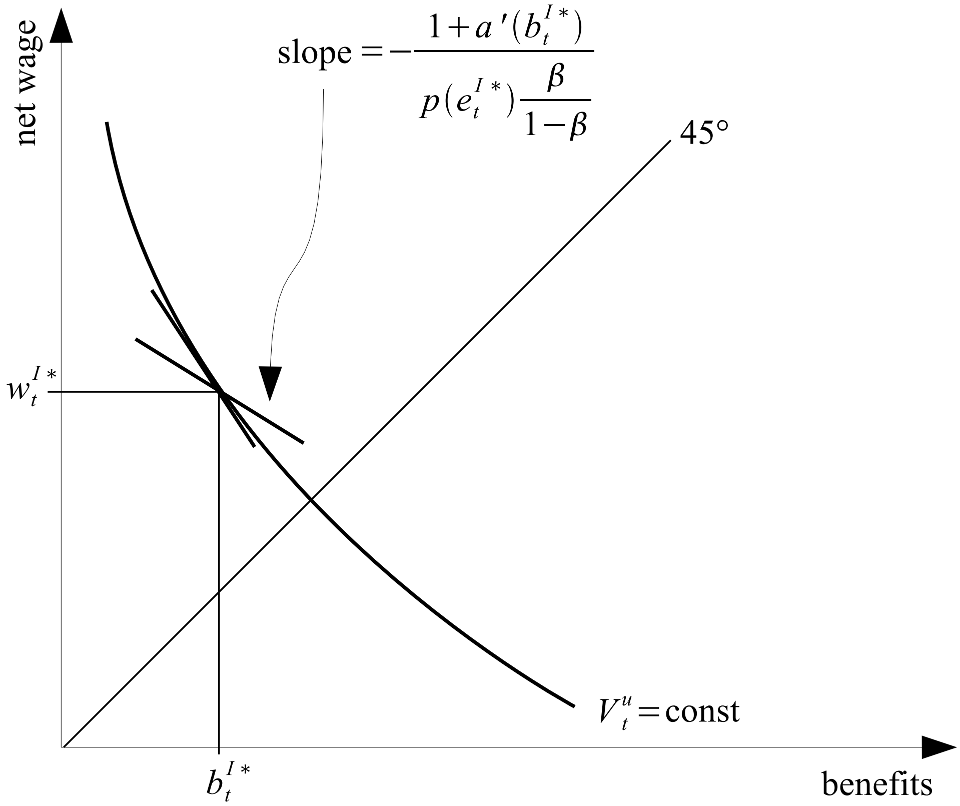

Figure 5: Illustrates assumption 1 in the model with benefit distribution costs under imperfect information.

6 Conclusion

In this chapter, I will summarize the results obtained, discuss shortcomings of the model and outline how to proceed.

I first studied an agent's fate who does not have access to UI and derived the autarky value. In order to improve an unemployed agent's well-being, the principal sets up a UI that promises the agent a utility level at least as high as the autarky value. Using the theory of recursive contracts, I presented the basic model by Hopenhayn and Nicolini and then extended it to allow for administrative costs. All variants of the basic model establish the basic results obtained by Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). The sequence of benefits decreases and the sequence of taxes increases. Although consumption is not fully smoothed, the agent is not worse off in terms of expected discounted utility. No matter what the UI system looks like, it always provides the agent with the same expected discounted utility. This means if the principal wants to finance benefit distribution costs, he can lower benefits "today", however, then he must raise after-tax income "tomorrow" in order to readjust utility. This is the trade-off as shown in figure 3, page 31-a, and figure 4, page 37-a. Furthermore, the analysis provided an upper bound for the amount of per-period tax. However, to establish the results in an environment with administrative costs under imperfect information, it was necessary to assume that the Lagrange multiplier on the incentive-compatibility constraint is positive. I argued that it does not make sense to assume the contrary. I gave a graphical illustration of the assumption that pointed out that providing benefits under imperfect information is more costly than under perfect information in terms of foregone wage income.

I will mention only one shortcoming of the model that the optimal UI literature has not mentioned and therefore studied so far. Other points of critique concern the stationarity of the probability function (see Shavell and Weiss (1979)), the undesirable immiseration result (see Pavoni (2003, 2006)) and the effect of UI on job termination on the side of the principal and the agent (see Feldstein (1976, 1978)).

The current model assumes an overly simple search theoretical environment in which only effort matters while abstracting from a reservation wage. Shavell and Weiss (1979) and Ljungqvist and Sargent (2004, exercise 21.3) include a reservation wage but without considering taxation. In the Shavell and Weiss model, the agent privately samples a gross wage from a publicly known wage distribution that he can positively influence by searching harder. He accepts a job-offer in form of the gross wage if and only if his after-tax wage is equal or larger than his reservation net wage. Determining a history-dependent tax that is based on an uncertain gross wage seems complicated. For every possible realization of the gross wage, the principal has to set up a tax system being contingent on the length of unemployment. Hence, for every possible tax system, there is different continuation value of employment. As a consequence, the principal can no longer directly control an employed agent's consumption. This limits the continuation value of employment's capability to serve as an incentive device. To sum up, it seems difficult to combine a history-dependent tax and a reservation wage because then we would have to work with taxation under uncertainty.

I will now turn to discussing what to do with my analysis. Hopenhayn and Nicolini (1997) solved a parametrized version numerically. In order to study administrative costs within this framework, we need to specify what the administration cost function looks like. We have to consider three items: Fixed costs, benefit distribution and tax collection costs.

Look at fixed costs first. Although we did not study them above, they matter in a quantitative analysis. Regardless of the UI agency's activity, an amount accrues every period that we can discount to the initial period. Introducing them then amounts to shifting the function $a(\cdot)$ upwards and specifying the point $a(0)$ which equals the discounted sum of fixed costs. To determine this value, we could look at financial statistics of actual UI systems and find the amount of fixed costs relative to per period expenditures on benefits or per period revenues. This gives a number $\kappa_1\%$. We then study the basic model without administrative costs and find the per period benefits and taxes and average them. We set per period fixed costs equal to $\kappa_1\%$ of this average. To sum up this proposal, per period fixed costs are given by the $\kappa_1\%$ of a UI's average transactions.

Consider now benefit distribution costs. We assume that other public authorities force the UI not let relative costs of distributing benefits, $a(b_t)/b_t$, rise beyond $\kappa_2\% < 100\%$. We first solve the model without benefit distribution costs and look for the highest value of b_t in the two informational settings and set $b^{\max} = \max\{b^*, b_0^*\}$. Then we set $a(b^{\max})/b^{\max} = \kappa_2/100$ and this specifies one point of benefit distribution cost function. The derivative $a'(b^{\max})$ must not necessarily equal $\kappa_2/100$. The value only determines the line of relative benefit distribution costs' slope.

Turn now to tax collection costs. As the analysis above indicated, the point where the amount of taxes collected satisfies $a'(W(V_t^e)) = -1$ gives an upper bound for the tax revenue. We could use this feature to simulate a legal constraint on taxation. Collecting taxes of more than $\kappa_3\%$ of the gross wage w could be ruled out. This means that after-tax income may not fall below $(100 - \kappa_3)\% \cdot w$. Thus, the maximum amount of discounted taxes collected is $-[(\kappa_3/100)w]/(1-\beta) =: W^{\max}$ and we set $a'(W^{\max}) = -1$. However, this has not yet specified the absolute value of $a(W^{\max})$. Similarily to above, we could assume that relative taxes $-a(W(V_t^e))/W(V_t^e)$ may not rise beyond $\kappa_4\%$. Hence, we set $a(W^{\max})/W^{\max} = -\kappa_4/100$.

It is left to specify the administration cost function's curvature. Hence, we would need to think about its third derivative. At present, I cannot think of reasons that justify $a'''(\cdot) > 0$ or $a'''(\cdot) < 0$. The question is its sign's significance.

Figure 6 on page 40-a summarizes the preceding story. $b^{\max}$ and $W^{\max}$ are most likely not the same. The figure inherits one assumption. Subsidizing work costs the same as distributing benefits. If we suspended with this assumption, we would have to specify an additional function for positive values different from the function $a(\cdot)$.

Given the precise specifications of the administration costs function, we can numerically solve the extended Hopenhayn and Nicolini model and find answers to the questions asked in the introduction. In particular, the numerical analysis will show whether the Lagrange multiplier assumption can be maintained. We can look how the replacement ratio, b_t/w, the consumption ratio w_t/w, the search effort and the probability of staying unemployed. Since the agent's expected discounted utility is the same in the model with administrative costs, benefits and after-tax income must somehow shift to finance them. The question is how much these shifts yield in terms of covering administrative costs. This gives rise to the question of the effect on total costs. Hopenhayn and Nicolini (1997) report total cost saving effects of introducing an optimal UI with taxation compared to an optimal UI without taxation and the current system. Looking at the three variants separately will help figure out the impact of the different cost structures. We could carry out a similar analysis under different assumptions about the administration cost function. I conjecture that the cost savings effect is reduced and the amount of the reduction is of interest. Building on the graphical analysis, I further propose studying the ratio b_t/w_t, because, for instance in the German UI, the amount of benefits is based on the net income (see §129 SGB III). Comparing the evolution of the ratio in the models without and with administration costs allows studying the coinsurance element due to the consideration of administrative costs. Depending on the administrative cost structures, welfare gains from introducing an optimal UI with taxation will be reduced. A quantitative analysis will provide answers about the reduction's extent.

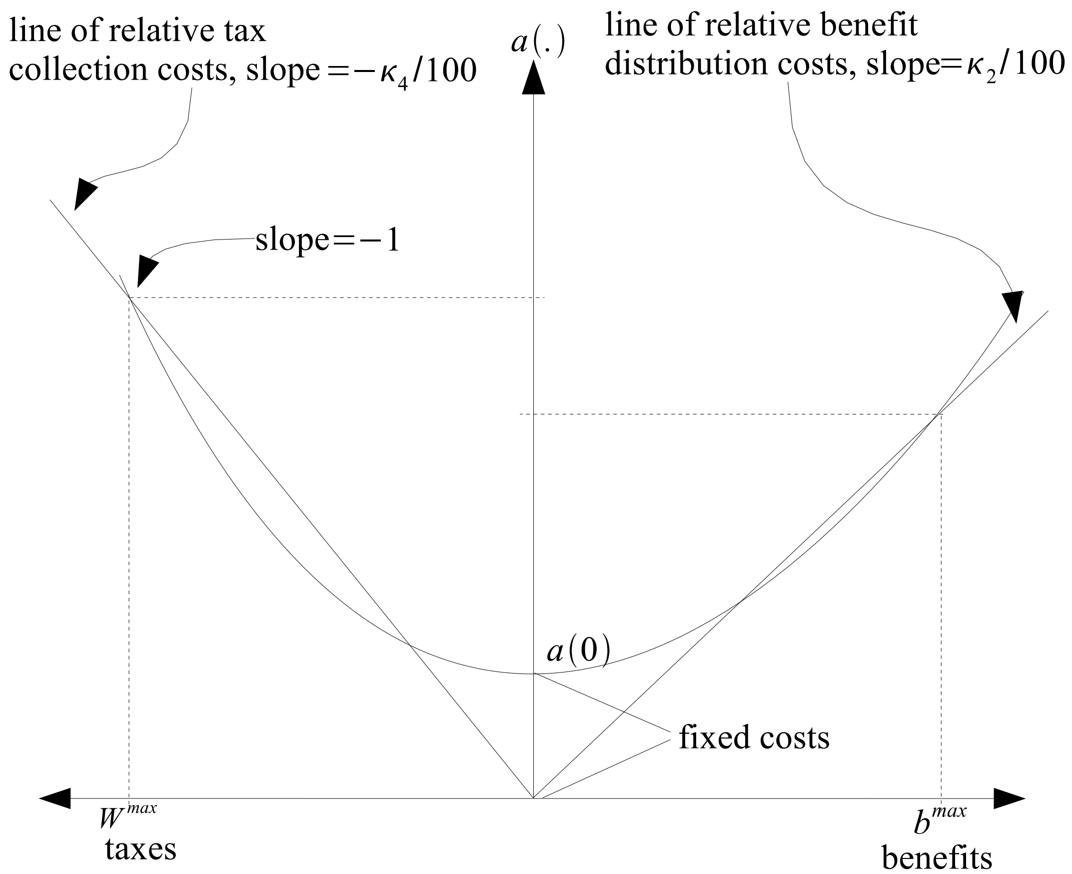

Figure 6: Illustrates the administration cost function $a(\cdot)$

A Derivation of the Derivatives of $\mathbf{W}(\mathbf{V_t^e})$

Restate the function $W(V_t^e)$:

$$W(V_t^e) = \frac{-w + u^{-1}\big((1-\beta)V_t^e\big)}{1-\beta}$$

The first derivative is:

$$\begin{aligned} W'(V_t^e) &= \frac{(u^{-1})'\big((1-\beta)V_t^e\big)}{1-\beta} \\ &= \frac{1-\beta}{(1-\beta)u'(w_t)} \\ &= \frac{1}{u'(w_t)} \end{aligned}$$

The second derivative is:

$$\begin{aligned} W''(V_t^e) &= \left(\frac{1}{u'\Big(u^{-1}\big[(1-\beta)V_t^e\big]\Big)}\right)' \\ &= -1u'(w_t)^{-2}u''(w_t)\frac{1}{u'(w_t)}(1-\beta) \\ &= \frac{-u''(w_t)(1-\beta)}{\big(u'(w_t)\big)^3} \end{aligned}$$

B Preliminaries for the Proof of Proposition 3

Condition (4.22) can be manipulated to yield:

$$C(V_{t+1}^u) - W(V_t^e) = -\eta_t \frac{p''(a_t)}{p'(a_t)}\big(V_t^e - V_{t+1}^u\big) \tag{B.1}$$

The envelope condition (4.28) can be manipulated to yield:

$$\eta_t = \big(W'(V_t^e) - C'(V_{t+1}^u)\big)\frac{\big(1-p(a_t)\big)p(a_t)}{p'(a_t)} \tag{B.2}$$

Substitute equation (B.2) into equation (B.1):

$$C(V_{t+1}^u) - W(V_t^e) = -\big(W'(V_t^e) - C'(V_{t+1}^u)\big)\frac{\big(1-p(a_t)\big)p(a_t)}{\big(p'(a_t)\big)^2}\frac{p''(a_t)}{p'(a_t)}\big(V_t^e - V_{t+1}^u\big) \tag{B.3}$$

The incentive-compatibility constraint (4.20) can be rearranged:

$$\left(V_t^e - V_{t+1}^u\right) = \frac{1}{\beta p'(a_t)} \tag{B.4}$$

Substitute equation (B.4) into equation (B.3) and we get condition (a):

$$C(V_{t+1}^u) - W(V_t^e) = \frac{-p''(a_t)\big(1-p(a_t)\big)p(a_t)}{\big(p'(a_t)\big)^3\beta}\big(W'(V_t^e) - C'(V_{t+1}^u)\big) \tag{B.5}$$

To derive condition (b), solve equation (B.2) for $\big(W'(V_t^e) - C'(V_{t+1}^u)\big)$ and replace in equation (B.5):

$$C(V_{t+1}^u) - W(V_t^e) = -\eta_t \frac{p''(a_t)}{\big(p'(a_t)\big)^2\beta} \tag{B.6}$$

C Derivation of Equation (5.15)

I restate the promise-keeping constraint (4.10) and manipulate it:

$$\begin{aligned}
& U(b_t) - e_t + \beta p(e_t)V_t^e + \beta(1-p(e_t))V_{t+1}^u = V_t^u \\
\Leftrightarrow\ & U(b_t) - e_t + \beta p(e_t)V_t^e - \beta V_t^e + \beta(1-p(e_t))V_{t+1}^u = V_t^u - \beta V_t^e \\
\Leftrightarrow\ & U(b_t) - e_t + \beta(1-p(e_t))\big(V_{t+1}^u - V_t^e\big) = V_t^u - \beta V_t^e \\
\Leftrightarrow\ & U(b_t) - e_t - (1-\beta)V_t^e + \beta(1-p(e_t))\big(V_{t+1}^u - V_t^e\big) = V_t^u - \beta V_t^e - (1-\beta)V_t^e \\
\Leftrightarrow\ & U(b_t) - U(w_t) - e_t + \beta(1-p(e_t))\big(V_{t+1}^u - V_t^e\big) = V_t^u - V_t^e \\
\Leftrightarrow\ & U(b_t) - U(w_t) - e_t + \beta p(e_t)\big(V_t^e - V_{t+1}^u\big) = V_t^u - V_t^e - \beta V_{t+1}^u + \beta V_t^e \\
\Leftrightarrow\ & U(b_t) - U(w_t) - e_t + \beta p(e_t)\big(V_t^e - V_{t+1}^u\big) \\
& \quad = (\beta-1)V_t^e + V_t^u - \beta V_{t+1}^u + V_{t+1}^u - V_{t+1}^u \\
\Leftrightarrow\ & U(b_t) - U(w_t) + \beta p(e_t)\big(V_t^e - V_{t+1}^u\big) - e_t = (\beta-1)\big(V_t^e - V_{t+1}^u\big) - \big(V_{t+1}^u - V_t^u\big)
\end{aligned}$$

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Erklärung

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Christoph Braun