University of Bonn - Department of Economics

Economic Theory Seminar

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Lecture No. 1

Price Dispersion and Advertising

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introduction

- analysis of a market that operates under imperfect information
- purpose of the paper: explain price dispersion
- mean of explanation: advertising
- market under perfect information
  $\rightsquigarrow$ unique price equilibrium
- market under imperfect information
  $\rightsquigarrow$ numerous price equilibria

create a model

- concept of equilibrium?
- equilibrium conditions that yield a non-trivial distribution of prices? not only a unique price.
- which parameters determine
  - level of prices?
  - degree of price dispersion?
- are there externalities when consumers search?
- welfare judgements of the model's outcome?

market concept

there are many types of markets:

- structure of information flows
- degree of centralization
- homogeneity of the good traded
- types of services provided jointly with the good
- number of actual and potential buyers
- volatility of parameters affecting supply and demand

- time span of the market
- geographical location of the market

starting point:

take market structure as given.

$\rightsquigarrow$ derive an equilibrium for this market.

warning: market structure itself is endogenous to a fuller model.

outline

- basic model: ads, no search
  - discrete time, unit demand for a homogenous good
  - many sellers and buyers. environment of perfect competition.
  - sellers set the price: monopolistic competition
- extension I: ads, and search
- efficiency of the market - welfare analysis
- extension II: continuous time
- extension III: oligopoly theory: number of firms is finite
- conclusion, objections, discussion

preliminary remarks - gedankenexperiment

if

- buyers know equilibrium price distribution.
- sellers know equilibrium price distribution.
- sellers maximize profits.
- buyers' search cost are larger than c.
- consumer preferences are well-behaved.

then there is at most one equilibrium price, the monopoly price.

proof. by contradiction assume in addition:

- consumers demand exactly one unit of the good.
- all consumers have the same maximum willingness to pay, m

suppose the lowest price is below the monopoly price.

the seller in question raises his price by $\varepsilon < c$.

$\Rightarrow$ he loses no customers. price hike is less than searching for a new seller.

repeated purchases: set ε s.t. the total increase in future expenditures is less than c.

$\Rightarrow$ revenue increases without affecting costs.

$\Rightarrow$ seller has not maximized profits.

a contradiction to the assumption that the seller is a profit maximizer. q.e.d.

how do we deal with this story?

change assumptions

- sellers do not know the price distribution.
  $\rightsquigarrow$ they believe that raising prices loses customers.

- buyers do not know the price distribution.
  $\rightsquigarrow$ customers buying at the lowest price believe that lower prices are available.
  $\Rightarrow$ a small price rise makes them search for a new seller.

- drop assumption of rationality.
  $\rightsquigarrow$ sellers subsidize buyers.
  $\rightsquigarrow$ in anger about a price rise customers search elsewhere.

- customers have zero search costs.
- or have some other source of information.

the following model uses the last two assumptions.

basic model - assumptions

- homogenous good is being traded for money.
- many buyers, M, and sellers, N
- sellers send ads: price and location
- sellers can advertise more than one price.
- time periods:
  1. set of new buyers enters the market.
  2. buyers receive ads.
  3. they buy one good or forfeit this opportunity.

- same maximum willingness to pay for all consumers, m.
  $\Rightarrow$ identical consumers

- expected cost of reaching a buyer with a single ad, b
  an ad always reaches a buyer.
  $\Rightarrow$ no economies of scale in advertising.

- ads are randomly allocated. each buyer has an equal chance of receiving each message.

- random allocation of ads is *independent* of the allocation of other ads.
  $\Rightarrow$ a buyer may receive more than one ad. it is not possible to send exactly one ad to one buyer.

- buyers receive ads free of charge.

- they cannot affect the probability of receiving an ad.

- they cannot obtain information about sellers.

- they cannot search for sellers.

- they cannot ask other buyers for recommendations.

- only consumers who receive at least one ad can buy a good. $\rightsquigarrow$ actual buyers.
  $\Rightarrow$ no ad, no purchase. $\rightsquigarrow$ potential buyers.

- actual buyers want to purchase the good from the seller who advertises the lowest price.
  $\rightsquigarrow$ vertical unit-demand up to the price m
  more than one lowest price? roll a die!

- homogeneity of consumers + unit-demand for the good
  $\Rightarrow$ no consumption distortion, who receives an, buys an ad (as long as the price is lower than or equal to m)

- they send an offer to the seller. transaction is costless.
  here the location in a spatial sense of a seller is irrelevant. it only matters in an informational sense.
  if a buyer has to face transaction costs, a consumption distortion might arise.
  $\rightsquigarrow$ possibly $p +$ transaction cost $> m$
  $\Rightarrow$ no purchase takes place

- a seller fulfills an order at the advertised price.

- constant cost of production p_0 per unit for all sellers.

- sellers produce upon receiving an order.
  $\rightsquigarrow$ horizontal supply at price p_0. $\rightsquigarrow$ no inventory costs
  $\rightsquigarrow$ abstract from production problems.

- all sellers know m.

- all sellers know the advertising price distribution.

- they do not know which ad is received by a specific buyer.

- sellers choose an advertising policy to maximize expected profits taking the behavior of other sellers as given.

definition 1 (advertising policy).
an advertising policy consists of the following:

- *price, or prices to advertise*

- *number of ads sent out at each price*

- *formally:* $\alpha : \mathbb{R}^+ \rightarrow \mathbb{Z}^+$
  $\alpha(p) = 2$ *means that at price* p *two ads are sent.*

- *mixed advertising stratgies:*
  probability measure μ *on the set of all functions* $\alpha : \mathbb{R}^+ \rightarrow \mathbb{Z}^+$

economic interest

- set of advertising strategies. $\rightsquigarrow$ variables of economic interest.
- market advertising price distribution gives the total number of ads sent out at each price.
- a random process determines the allocation of ads among potential buyers.

$\rightsquigarrow$ the combination of market advertising price distribution and random process gives the sales price distribution.

$\rightsquigarrow$ the sales price distribution specifies the number of sales at each price.

$\rightsquigarrow$the advertising strategy and the random process determine the expected profit of a seller.

is there an equilibrium of advertising strategies?

equilibrium concept?

definition 2 (nash equilibrium of advertising strategies).
no seller can increase his expected profit by changing his strategy while all other sellers maintain their strategies.

we want to know:

- equilibrium distribution of advertising prices
- equilibrium distribution of sales prices
- properties of these distributions

methodical notes I

- different combinations of M and N, different set of nash equilibria.

- restrict sellers to fixed strategies. $\Rightarrow$ no nash equilibrium exists.

how to get around this problem?

- allow mixed strategies.
  $\Rightarrow$ at least one nash equilibrium exists.

- maintain fixed strategies and introduce a different equilibrium concept.

definition 3 (nash-ε-equilibrium).
no seller can increase his expected profit per advertising message by more than ε.

proposition 1.
there exists N^* such that for all $N \geqslant N^*$, and all M, ε-equilibria exist.

- ε can be interpreted as a stability parameter in the market.
- ε small, market is more stable.

the larger N, the smaller ε? as a result of fiercer competition?

methodical notes II

- analysis for finite M, and N is painfully complicated.

- makes life easy: $M \to \infty$, $N \to \infty$
  $\Rightarrow$ convergence of the equilibrium price distributions to a limit that can be easily derived, explained, and grasped.

- for large M, and N (still finite) the analysis can be done as if $M \to \infty$, $N \to \infty$

- use the infinity case to approximate the finite case.

- is this a valid approach?
  $\rightsquigarrow$ appendix a

random process - the urn model

- urn, and balls correspond to buyers' mailboxes and ads.
- n urns, r balls
- probability of dropping a ball in any urn is $1/n$
- drop one ball.
  process is a bernoulli-experiment with $X \sim B(1, 1/n)$
- $B(r, p)$: binominal distribution with r trials and probability of success p.
- drop r balls, i.e. repeat the bernoulli-experiment r-times.
  process is a bernoulli-experiment with $X \sim B(r, 1/n)$

- binominal-distribution can be approximated by a poisson-distribution.

proposition 2.

let X be distributed as $B(n,p)$, a binomial random variable with parameters n and p. Suppose

$$\lim_{n\to\infty} np = \lambda,$$

or

$$np = \lambda = const,$$

where λ is a positive real constant, then X is asymptotically distributed as $P(\lambda)$, a poisson distribution with parameter λ.
put briefly,

$$\lim_{n\to\infty} P(X=m) = \frac{\lambda^m}{m!}e^{-\lambda}$$

proof. Let $X \sim B(n,p)$. So

$$
\begin{aligned}
P(X=m) &= \frac{n!}{m!(n-m)!}p^m(1-p)^{n-m} \\
&= \frac{n!}{m!(n-m)!}\left(\frac{\lambda}{n}\right)^m\left(1-\frac{\lambda}{n}\right)^{n-m} \\
&= \frac{n!}{n^m(n-m)!}\frac{\lambda^m}{m!}\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-m} \\
&= \frac{\lambda^m}{m!}\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-m}\frac{n(n-1)(n-2)\cdots(n-m+1)}{n^m} \\
&= \frac{\lambda^m}{m!}\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-m}\frac{n^m\left\{1\cdot(1-1/n)\cdot\ldots\cdot\left(1-\frac{m-1}{n}\right)\right\}}{n^m} \\
&= \frac{\lambda^m}{m!}\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-m}\left\{1\cdot(1-1/n)\cdot\ldots\cdot\left(1-\frac{m-1}{n}\right)\right\}
\end{aligned}
$$

Look at the expressions where n appears:

$$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n = e^{-\lambda}$$

$$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n = 1$$

$$\lim_{n\to\infty}\frac{1\cdot\left(1-\frac{1}{n}\right)\cdot\ldots\cdot\left(1-\frac{m-1}{n}\right)}{\left(1-\frac{\lambda}{n}\right)^m} = 1$$

Hence,

$$\lim_{n\to\infty} P(X=m) = \frac{\lambda^m}{m!}e^{-\lambda}$$

q.e.d.

application to our problem

- number of trials $=$ number of ads, probability of success $= 1/M$
  $\rightsquigarrow r = \sharp$ads, $p = 1/M$

- the mean of the binomial distribution may not change.
  $\rightsquigarrow$ as the number of buyers increases, $M \nearrow$, the number of ads also has to increase such that the ratio remains constant.
  $\rightsquigarrow \lambda =$ number of ads / M

- probability that an urn is empty corresponds to the probability that a buyer does not receive any ads,

$$\Rightarrow P(X = 0) = e^{-(\sharp \text{ads})/M}$$

first results

any price p advertised has to cover production and advertising costs $p_0 + b$

proposition 3.
$p = p_0 + b$ *is not an equilibrium.*

proof. • several ads might go to the same buyer.

- not every ad triggers a sale.
- $b = p - p_0$, revenue minus cost, net of production cost
- advertising cost per sale, $\hat{b}$, is larger than b.
- thus, $p - p_0 - \hat{b} < 0$

- the expected profit per item sold is negative.
- $p = p_0 + b$ cannot be an equilibrium.

q.e.d.

recall:

- perfect competition drives the price down to marginal costs (here: $p_0 + b$).
- result does not hold.

proposition 4.
$p^* > p_0 + b$ is not an equilibrium.

proof. • offering a lower price may be profitable for one seller.

- consider a seller who cuts his price by $\varepsilon > 0$
- his customers stay with him.
- he might get customers from some other seller who offers the same price.
- mark-up profit per item sold, $p^* - \varepsilon - p_0$
- the mark-up profit per additional customer exceeds the revenue loss due to a lower price as long as ε is small.

q.e.d.

proposition 5.
not more than one seller can advertise any given price in equilibrium.

proof. • suppose two sellers were advertising the same price.

- now, one seller cuts his price.

- use the argument above.

- increase in expected profit due to an increase in expected sales is larger than the decrease in revenue due to a lower price.

q.e.d.

discussion of the results so far

- monopolistic competition does not drive down the price to marginal total cost.
- all sellers offer different prices. no unique price equilibrium.
- two classical results of perfect competition do not hold.
  $\rightsquigarrow$ imperfect competition.

definitions

definition 4.
$A(p)$ denotes the expected number of ads sent out at prices less than or equal to p divided by the number of buyers, M.

- usually $A(m) < 1$, not every buyer receives an ad
- call $A(p)$ the advertising price distribution function.
- $A(p)$ increases, more competition for buyers.

definition 5.
the advertising price density function is defined as $a(p) = A'(p)$. moreover,

$$a(p)\Delta p = \frac{\textit{expected number of ads at prices } p^* \in [p, p + \Delta p]}{M}$$

definition 6.
$S(p)$ denotes the expected number of sales at prices less than or equal to p divided by the number of buyers, M.

also note that $S(m) < 1$, i.e. not every buyer buys. nevertheless, call $S(p)$ the sales price distribution function.

definition 7.
the sales price density function is defined as $s(p) = S'(p)$. moreover,

$$s(p)\Delta p = \frac{\textit{expected number of sales at prices } p^* \in [p, p + \Delta p]}{M}$$

definition 8.
p_{min}, and p_{max} denote the minimum and maximum price at which ads are ever sent, respectively.

definition 9.
$\pi(p)$ denotes the probability that a given ad at price p results in a sale.

- $\pi(p)$ is a kind of downward-sloping demand function.
- this probability is then equal to the expected number of sales at a given price divided by the number of expected ads at a given price.
- if Δp becomes infinitesimally small, we may write

$$\pi(p) = \frac{s(p)}{a(p)}$$

which is equivalent to

$$s(p) = \pi(p)a(p).$$

- as the number of buyers, M, increases, the probability that a buyer receives more than two ads tends to zero. then each additional ad triggers a sale.
  $\rightsquigarrow \pi(p)$ is the probability that an additional ad results in one more sale.

definition 10.
set up the following variable:

$$P(p) = (p - p_0)\pi(p) - b$$

in words, (revenue minus production costs) minus advertising costs. expected profit per item sold.

if we sum up over all sales at prices p, we get the following:

$$\begin{aligned}\text{total profit} &= \int [(p-p_0)s(p) - ba(p)]dp \\ &= \int [(p-p_0)\pi(p)a(p) - ba(p)]dp \\ &= \int [(p-p_0)\pi(p) - b]a(p)dp \\ &= \int P(p)a(p)dp\end{aligned}$$

in the limit, $M \to \infty$, $N \to \infty$, $P(p)$ is the *marginal product of advertising*

in the limit, an additional ad triggers a sale, see note above.

probability of displacing an ad of his own approaches zero.
$\Rightarrow$ if a seller changes his advertising strategy, an expected loss in profit due to less expected sales is negligible.

definition 11.
as M, and N get large, the functions A, S, π, P, and p_{min} converge to a limit. denote these limits by A^*, S^*, π^*, P^*, and p^*_{min}

main results of the basic model

proposition 6.
for all p holds: $P^*(p) = 0$, *thus* $\pi^*(p) = b/(p - p_0)$

proof. • within $\varepsilon > 0$ $P(p)$ is a good approximation for the marginal product of advertising.

- if $P(p) \geqslant \varepsilon > 0$, a seller sends out one more ad and increases his profit.

- if $P(p) \leqslant -\varepsilon < 0$, a seller reduces the number of ads and thus increases his profit.

- in equilibrium, it must hold for all p: $\mid P(p) \mid < \varepsilon$

- in the limit, $P^*(p)$=0.

- manipulate this expression:

$$
\begin{aligned}
|\ P(p)\ | &= |\ (p-p_0)\pi(p) - b\ | \\
&= \left|(p-p_0)\left(\pi(b) - \frac{b}{p-p_0}\right)\right| \\
&= (p-p_0)\left|\pi(p) - \frac{b}{p-p_0}\right| < \varepsilon
\end{aligned}
$$

this expression must hold for all p, especially m. therefore,

$$\begin{aligned}
&\Rightarrow \quad (m - p_0)\left|\pi(p) - \frac{b}{p - p_0}\right| < \varepsilon \\
&\Leftrightarrow \quad \left|\pi(p) - \frac{b}{p - p_0}\right| < \frac{\varepsilon}{m - p_0} \\
&\Rightarrow \quad \pi^*(p) - \frac{b}{p - p_0} = 0 \\
&\Leftrightarrow \quad \pi^*(p) = \frac{b}{p - p_0}
\end{aligned}$$

q.e.d.

when the number of buyers and sellers is large, the marginal product of advertising is zero.

moreover, the expected profit per item sold is zero.

$\rightsquigarrow$ the expected profit of a monopolistic competitor is zero. zero-profit condition.

proposition 7.
all prices $p \in (p^*_{min}, m)$ *are advertised in equilibrium.* $p^*_{min} = p_0 + b$.

proof. • suppose there is a price range $(p_1, p_2) \subseteq (p_{\text{min}}, m)$ in which no prices are advertised.

• a seller advertising p_1 loses no customers when he now advertises p_2.

• his revenue per sale increases by $p_2 - p_1$, his number of sales remains the same.

- costs have not changed, thus the profit has risen.

- a contradiction, this cannot be an equilibrium where there is a range with no prices advertised.

next part of the proof.

- it holds that $\pi(p_{\text{min}}) = 1$. the lowest price possible always triggers a sale.

- otherwise an ad at this price would lose money.

- make use of proposition 6:

$$
\begin{aligned}
|P(p_{\text{min}})| &= |(p_{\text{min}} - p_0)\pi(p_{\text{min}}) - b| \\
&= |p_{\text{min}} - (p_0 + b)| < \varepsilon
\end{aligned}
$$

- M, and N get larger
  $\Rightarrow p^*_{\text{min}} = p_0 + b$

q.e.d.

proposition 8.
for all p it holds:

$$\pi^*(p) = e^{-A^*(p)}$$

proof. • for large M π is the probability than an additional ad triggers a sale.

- thus this is the probability that the ad goes to a customer who has not received an ad advertising a price less than or equal to p.

- in a sense his urn is empty.

- the probability that an urn is empty is given by exp($-$balls/urns).
- in this case then $\pi(p) = \exp(-A(p))$.
- in the limit, $\pi^*(p) = \exp(-A^*(p))$.

q.e.d.

note:

- $\pi^*(m)$ is the probability that an ad offering price m results in a sale.
- as seen above, $\pi^*(m)$ is the probability that a buyer has not received an ad offering a price lower than or equal to m. his mailbox is empty.
- the probability that a buyer does not receive an ad is equal to the probability that an ad offering m triggers a sale.

proposition 9.
equilibrium distributions of sales and advertising prices are given by:

$$A^*(p) = \begin{cases} 0 & p \leqslant p_0 + b \\ \ln((p - p_0)/b) & p_0 + b \leqslant p \leqslant m \\ \ln((m - p_0)/b) & p \geqslant m \end{cases}$$

$$a^*(p) = \begin{cases} 0 & p \leqslant p_0 + b \\ 1/(p - p_0) & p_0 + b \leqslant p \leqslant m \\ 0 & p \geqslant m \end{cases}$$

and

$$s^*(p) = \begin{cases} 0 & p \leqslant p_0 + b \\ b/(p - p_0)^2 & p_0 + b \leqslant p \leqslant m \\ 0 & p \geqslant m \end{cases}$$

$$S^*(p) = \begin{cases} 0 & p \leqslant p_0 + b \\ 1 - b/(p - p_0) & p_0 + b \leqslant p \leqslant m \\ 1 - b/(m - p_0) & p \geqslant m \end{cases}$$

proof. • from propositions 6, and 8 immediately follows:

$$A^*(p) = \ln\left(\frac{p - p_0}{b}\right)$$

- proposition 7 gives the price ranges.

- prices less than $p_0 + b$ are not advertised, $A^*(p) = 0$.

- for prices above m the number of ads does not increase, $A^*(p) = A^*(m) = \ln((m - p_0)/b)$.

- differentiating $A^*(p)$ gives $a^*(p)$.

- $s^*(p)$ is calculated using proposition 6 and using the relationship $s^*(p) = \pi^*(p)a^*(p)$.

- integrating this expression yields $S^*(p)$.

- exploiting the fact the the cumulative distribution function is continuous from the right gives the last entry for $S^*(p)$.

q.e.d.

discussion of the results so far

- the functions above only describe market behavior.
- they do not tell how a specific seller acts in equilibrium.
- sellers' strategies are indeterminate.
- if we restrict these strategies, the propositions still hold.
- example: advertising only one price is permitted.

summary statistics and comparative statics

define the following variables:

- $d = p_{\mathsf{max}} - p_0$, the maximum revenue per item sold, or the maximum profit margin.
- $\overline{p}$: mean of the sales price distribution
- var(p): variance of the sales price distribution, can be regarded as a measure of the price dispersion in the market.

one can show:

$$S^*(m) = (d-b)/d \tag{1}$$

$$\overline{p} = p_0 + \frac{bd}{b-d} \ln\left(\frac{d}{b}\right) \tag{2}$$

$$\mathsf{var}(p) = b(d-b) - \frac{b^2 d^2}{(d-b)^2} \ln^2\left(\frac{d}{b}\right) \tag{3}$$

$$A^*(m) = \ln\left(\frac{d}{b}\right) \tag{4}$$

for comparative statics we need the derivatives of the expressions above:

$$\frac{\partial S^*(m)}{\partial b} = -\frac{1}{d} < 0 \tag{5}$$

$$\frac{\partial \overline{p}}{\partial b} = \frac{d^2}{(d-b)^2} \ln\left(\frac{d}{b}\right) - \left(\frac{d}{d-b}\right) > \frac{1}{2} \tag{6}$$

$$\frac{\partial \mathsf{var}(p)}{\partial b} = d - 2b - \frac{2bd^2}{(d-b)^2} \ln\left(\frac{d}{b}\right)\left(1 + \frac{d}{d-b} \ln\left(\frac{d}{b}\right)\right) \tag{7}$$

$$\frac{\partial A^*(m)}{\partial b} = -\frac{1}{b} < 0 \tag{8}$$

put into elasticity form:

$$\frac{ES^*(m)}{Eb} = -\frac{b}{d-b} \in (-1, 0) \tag{9}$$

$$\frac{E(\overline{p} - p_0)}{Eb} = \frac{d}{d-b} - \frac{1}{\ln(d/b)} \in (\frac{1}{2}, 1) \tag{10}$$

$$\frac{EA^*(m)}{Eb} = -\frac{1}{\ln(d/b)} < 0 \tag{11}$$

$$\frac{ES^*(m)}{Ed} = -\frac{ES^*(m)}{Eb} \in (0, 1) \tag{12}$$

$$\frac{E(\overline{p} - p_0)}{Ed} = 1 - \frac{E(\overline{p} - p_0)}{Eb} \in \left(0, \frac{1}{2}\right) \tag{13}$$

$$\frac{EA^*(m)}{Ed} = -\frac{EA^*(m)}{Eb} > 0 \tag{14}$$

put in words:

- cheaper ads bring about a lower average price advertised. how the price dispersion varies is undeterminate.

- a higher maximum willingness to pay means that the average price advertised increases. there are more ads, and more sales. price dispersion also rises.

- if advertising is costless, there are so many ads that every one triggers a sale. the average price advertised then is p_0.

- if the cost of advertising is as large as the maximum profit margin d, no seller sends ads. thus there are no sales.

- a rising d may be due to a higher maximum willingness to pay or due to decreasing production costs.

- if the maximum profit margin increases, $S^*(m)$ tends to one, every ad triggers a sale. the average price advertised increases.

- then there are is an infinite number of ads. each consumer has an infinite number of ads.

- then receiving one more ad yields an expected benefit that tends to zero.

- mean of the advertising price distribution rises.

extension I: buyer's search

- now buyers can search, and have search costs that depend on the number N of firms in the market.
- $c(N)$, search cost function, $c'(N) \geqslant 0$

how do we model search?

- suggestion 1: equal probability of searching any given store.
- suggestion 2: probability of locating a seller is proportional to his sales.

the model adopts the following assumption:

assumption 1.
in addition, assume the following:

- *all buyers have identical cost of search functions* $c(N)$ *with* $c'(N) \geqslant 0$
- *a buyers waits for an ad. if he does not receive one, he starts searching for a seller.*
- *probability of receiving an offer from a particular seller his proportional to his sales.*
- *probability that this offer is within a certain price range is proportional to the share of sales within this price range.*
- *this means, a seller does not take advantage of a searcher*

a buyer has to do the following:

- if he receives at least one ad, he buys the good from the seller advertising the lowest price, or he starts searching for an even lower price
- if he does not receive any ads, he starts searching.
- he buys from the first shop he finds.

we may say a buyer has received a price offer $p \geqslant 0$.

he considers a search decision as follows:

- before search, he has received an offer $p \geqslant 0$.
- when he searches he finds a price offer $\hat{p}$.

- if he rejects this price, he has to pay $p + c(N)$.
- thus the accepts the price $\hat{p}$ if

$$\hat{p} \leqslant p + c(N)$$

- he does not accept prices higher than a cut-off price given by:

$$\begin{aligned} & p_c && \geqslant p + c(N) \\ \Leftrightarrow \quad & p_c - p && \geqslant c(N) \end{aligned}$$

  expected value of search greater than search costs

since all consumers are identical, it follows that there are no sales for prices $p_c \geqslant p_{\max}$.

furthermore, it cannot be true that $p_{\text{max}} < p < p_c$.

- suppose the number of ads for a seller with $p = p_{\text{max}}$, and for a seller with $p_{\text{max}} < p < p_c$ is the same.
- both have the same costs.
- both have the same number of expected sales.
- but the latter seller has a higher revenue due to a higher price.
- this cannot be an equilibrium.
- hence, $p_{\text{max}} = p_c$

to reiterate,

- implicitly assuming that $p_{\text{max}} < m$, they do not accept prices $p + c(N) \geqslant m$.
- if a buyer has not received an ad, he knows that he will find a seller offering a price at most as high as $p_c + c(N) = m$.
- hence, he starts searching.
- the converse also holds. if a buyer searches, we may conclude that he has not received any ads.
- for the price $\hat{p}$ he finds it holds $\hat{p} + c(N) \leqslant m$.
- then they will accept the first price they find.

more defintions

definition 12.
let ϕ denote the proportion of buyers who receive at least one ad. hence, the proportion $1 - \phi$ of consumers searches.

from what we have said above we conclude:

- ϕ is the proportion of sales due to advertising.
- $1 - \phi$ is the proportion of sales due to search.

above we have seen that $\pi(p) = s(p)/a(p)$.

definition 13.
let $\hat{\pi}$ denote the probability that a sales is directly due to an ad.

there is a relationship between $\hat{\pi}$ and π.

$$\begin{aligned}
\phi &= \text{proportion of consumers that receive ads} \\
&= \frac{\text{sales directly due to ads at price } p}{\text{expected number of sales at } p} \\
&= \frac{\text{sales directly due to ads at price } p/a(p)}{\text{expected number of sales at price } p/a(p)} \\
&= \frac{\text{sales directly due to ads at price } p/a(p)}{s(p)/a(p)} \\
&= \frac{\hat{\pi}(p)}{\pi(p)}
\end{aligned}$$

hence, $\phi\pi(p) = \hat{\pi}(p)$

in the basic model we had $\hat{\pi} = \pi$ since all sales were due to ads.

results

how do the results obtained in the previous section change?

proposition 10.
proposition 6 still holds.
from $\hat{\pi}^*(p_{min}) = 1$ *follows* $\pi(p_{min}) = 1/\phi$.

proposition 11.
we have:

$$\begin{aligned} P(p_{min}) &= (p_{min} - p_0)\pi(p_{min}) - b = 0 \\ \Leftrightarrow \quad (p_{min} - p_0)\frac{1}{\phi} &= b \\ \Leftrightarrow \quad p_{min} &= b\phi + p_0 \end{aligned}$$

proposition 12.
the probability that an ad goes to a buyer who has not received an ad with a price less than or equal to p is given by:

$$\hat{\pi}(p) = exp\{-A(p)\}$$

from this follows with help from the propositions and results above

$$\begin{aligned} A(p) &= -\ln \hat{\pi}(p) \\ &= -\ln \phi\pi(p) \\ &= -\ln \left(\phi \frac{b}{p - p_0} \right) \\ &= \ln \left(\frac{p - p_0}{\phi b} \right) \end{aligned}$$

proposition 13.

$$a(p) = \begin{cases} 0 & p \leqslant p_0 + b\phi \\ \frac{1}{p-p_0} & p_0 + b\phi \leqslant p \leqslant p_{max} \\ 0 & p \geqslant p_{max} \end{cases}$$

$s(p) = \pi(p)a(p),\ and\ \pi(p) = b/(p-p_0)\ yield$

$$s(p) = \begin{cases} 0 & p \leqslant p_0 + b\phi \\ \frac{b}{(p-p_0)^2} & p_0 + b\phi \leqslant p \leqslant p_{max} \\ 0 & p \geqslant p_{max} \end{cases}$$

we already know

$$p_{\mathsf{min}} = p_0 + b\phi \text{ ,and}$$

$$d = p_{\mathsf{max}} - p_0$$

next to this we want to know ϕ, and p_{max}.

with search all buyers buy (either via ads or search). thus,

$$\int_{p_{\mathsf{min}}}^{p_{\mathsf{max}}} s(p)dp = 1$$

integrating yields

$$\frac{1}{\phi} - \frac{b}{d} = 1$$

solving for ϕ gives

$$\phi = \frac{d}{b+d}.$$

at the cut-off price p_c the expected gain from search must equal the cost of search. the gain from search is $p_{\mathsf{max}} - p$. the expected gain is obtained by weighting this expression with the sales price density function $s(p)$. thus,

$$c = \int_{p_{\mathsf{min}}}^{p_{\mathsf{max}}} (p_{\mathsf{max}} - p) \frac{b}{(p - p_0)^2} dp$$

solving this integral gives:

$$c = b \left\{ \frac{\phi}{1 - \phi} - \ln \left(\frac{1}{1 - \phi} \right) \right\} \tag{15}$$

p_{max} can be written as:

$$p_{\mathsf{max}} = p_0 + d$$

as in the previous section it may be of interest to calculate:

- $\overline{p}$, the mean of the sales price distribution

- var(p), the variance of the sales price distribution

- $R(p) = p_{\text{max}} - p_{\text{min}}$, the maximum range of prices

- $A(p_{\text{max}})$, the average number of ads per buyer.

equation 15 implicitly defines d. it is not possible to solve for d explicitly.

however, for comparative statics this does not matter (use the implicit function theorem).

welfare analysis

is the amount of advertising and search efficient?

approach:

- set up the social planners problem and solve for the efficient amount of advertising and search.
- compare the results to the basic model without search.
- compare the result to the extended model with search.

welfare analysis for the basic model

social welfare is composed of

- consumers' surplus
- producers' profits $(= 0)$

for convenience, we measure social welfare as the average welfare gain per buyer.
$\rightsquigarrow$ valid approach since all buyers and sellers are identical.

average welfare equals

- value of an item
- minus production cost
- minus advertising cost

more formally,

- ξ is the proportion of consumers who receive ads.
  $\rightsquigarrow$ the average gain is ξm
- average production cost, ξp_0
  average advertising cost, $bA(m)$

thus,

$$
\begin{aligned}
& W && = \xi m - \xi p_0 - bA(m) \\
\Leftrightarrow\ & W && = \xi(m - p_0) - bA(m) \\
\Leftrightarrow\ & W && = S(m)(m - p_0) - bA(m) \\
\Leftrightarrow\ & W && = [(d - b)/d]d - b\ln(d/b) \\
\Leftrightarrow\ & W && = d - b - b\ln(d/b)
\end{aligned}
$$

this is the social welfare function in the free market.

a social planner asks whether there is an $A \neq A(m)$ that maximizes social welfare.

social planner

seek an A that maximizes social welfare.

from the urn model developed above we know that ξ is given by $1-\exp(-A)$.

- the probability that a buyer does not receive an ad is given by $\exp(-A)$
- thus, the proportion of buyers that obtain an item is given by $1-\exp(-A)$
- hence, $\xi = 1 - \exp(-A)$

social planner's problem:

$$\max_A W = (1 - e^{(-A)})d - bA$$

f.o.c.

$$\begin{aligned} \frac{\partial W}{\partial A} &= e^{(-A)}d - b = 0 \\ \Leftrightarrow A^* &= \ln(d/b) = A(m) \end{aligned}$$

social planner's problem and the free market's result coincide

the free market generates an efficient amount of advertising.

welfare analysis of the extended model

suppose all consumers buy

- a proportion $1 - e^{(-A)}$ of sales is due to advertising.
- a proportion $e^{(-A)}$ of sales is due to search

independent of whether a deal is due to ads or due to search, benefits per buyer and production costs per item remain constant.

$\Rightarrow$ to maximize social welfare minimize advertising and search cost c.

total costs are given by

$$C = bA + c\,\mathrm{exp}(-A)$$

the social planner has to choose an optimal A.

f.o.c.

$$\begin{aligned} & b + ce^{-A}(-1) = 0 \\ \Leftrightarrow \quad & e^{-A} = b/c \\ \Leftrightarrow \quad & A^* = \ln(c/b) \end{aligned}$$

if $c < b$, $A^* = 0$, only search is optimal.

if $c \geqslant b$, minimum costs C_{min} are given by:

$$b \ln(c/b) + b$$

in the search model, ϕ is the proportion of consumers who buy due to advertising.

$\Rightarrow$ a proportion $1 - \phi$ buys due to search.
$\Rightarrow 1 - \phi = \exp(-A) = b/c$

a social planner asks: in the case $c \leqslant b$, is the amount of search $\psi = 1 - \phi$ sufficient to maximize social welfare?

in this case we have $\psi < 1$, but in the free market we also have $A > 0$.

in the free market, we have seen that

$$\frac{c}{b} = \frac{\phi}{1 - \phi} - \ln\left(\frac{1}{1 - \phi}\right)$$

which has been derived from the fact that at the cut-off price the expected gain from search has to equal search costs.

manipulating this expression gives

$$\frac{c}{b} = \frac{1-\psi}{\psi} - \ln\left(\frac{1}{\psi}\right)$$

above we have seen that the optimal amount of search is given by

$$\frac{c}{b} = \frac{1}{1-\phi} = \frac{1}{\psi_0}$$

equating both equations gives

$$\frac{1}{\psi_0} = \frac{1-\psi}{\psi} + \ln\psi$$

we now have to compare ψ and ψ_0. manipulating the expression above yields

$$\psi_0 = \frac{\psi}{1 - \psi + \psi \ln \psi}$$

$\psi < 1$ implies $\psi_0 > \psi$

$\Rightarrow$ the free market brings about an insufficient amount of search.

are there ways to achieve an efficient operation of the market?

efficiency in the market

the efficient amount of advertising is given by

$$A^* = \ln(c/b)$$

the amount of advertising generated by the market is given by proposition 12

$$A(p_{\text{max}}) = \frac{p_{\text{max}} - p_0 - c}{b}$$

solutions:

- restrict p_{max} s.t.

$$A(p_{\text{max}}) = A^*$$

- impose a tax x on ads s.t.

$$A(p_{\text{max}}(b + x, c)) = A^*$$

- give a search subsidy y to consumers s.t.

$$A(p_{\mathsf{max}}(b, c - y)) = A^*$$

- the government takes over and provides the good.

if the proposals themselves involve costs, the analysis is incomplete.

advertising and competition

- if b rises, prices approach m
- sellers advertise less, it becomes too expensive
- however, in the free market there are too many ads.
- if prices approach m, nobody searches.
- then advertising is essential for the market not to break down.

specific prices in the model

for the welfare analysis above the specific price is irrelevant.

social welfare only depends on whether somebody is able to buy a good. the price he pays does not matter.

a social planner only determines how many buyers should obtain the good and the chooses the most efficient way to transport information about the availability of the good.